Financially Constrained Carbon Management[*]

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Abstract

We develop a model studying how financing frictions affect a firm's carbon footprint and its transition to sustainable technologies while allowing for multiple types of green investment: abatement of carbon emissions, adoption of available technologies, and green innovation. Financing frictions impact each type of green investment differently—with abatement unaffected, a negative effect on adoption, and an ambiguous impact on green innovation. Financing frictions reduce current emissions by contracting production, but have a negative impact on the transition to greener technologies in firms relying mainly on adoption. We further show that tilting strategies need not boost green innovation, exclusion strategies mainly curb current emissions, and subsidies to adoption also help incentivize green innovation.

Keywords: Carbon emissions; green investment; green transition; financing frictions.

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1 Introduction

The global need to reduce carbon emissions requires a massive amount of financial resources. In the remarkable effort to meet the Paris agreement's goal of limiting global warming, fiscal policies and private investors alike are contemplating programs aimed at channeling resources to the corporate sector. However, a preliminary question lies at the forefront of the global rush to green the economy: How do financial constraints (and the relaxation of such constraints) affect the firm's incentives to reduce their carbon emissions and to transition to more sustainable industrial processes? To date, works attempting to answer this question have overlooked that firms' carbon emission management is both dynamic and multifaceted: Firms have several tools at their disposal to best manage their carbon footprint over time and become technologically sustainable. This paper seeks to fill this gap.

Indeed, an assessment of the effects of financing frictions on corporate policies needs to take into account that: (1) a firm's carbon management policy is dynamic in nature and is affected by the availability of financial slack, which is itself an endogenous outcome; (2) a firm's *current* carbon emissions and its *long-term* carbon footprint depend on both production and green investment choices; and (3) green investment is heterogeneous: Firms can target emission reduction in the short term without changing the firm's production technology (by simply offsetting their current emissions) or instead promote a long-term transition by investing in sustainable technologies and, thus, greening their production process. Our framework takes all of these aspects into account.

Specifically, we develop a model to investigate how financing frictions affect the different margins through which firms can simultaneously reduce their carbon emissions and make their technology more sustainable. Financing frictions are modeled as costs incurred when raising outside financing, which make it optimal for the firm to accumulate cash reserves as in Décamps, Mariotti, Rochet, and Villeneuve (2011) or Bolton, Chen, and Wang (2011). We assume that the firm operates in a jurisdiction where carbon emissions are priced and customers value a firm's sustainability (see, e.g. Derrien, Krueger, Landier, and Yao, 2023;

Meier, Servaes, Wei, and Xiao, 2023). In this setting, we study the real and financial policies that maximize shareholder value.

A notable feature of our setting is that we discriminate among different types of green investment by their impact on the firm's technological sustainability. At one extreme of the spectrum, we can think of abatement, which aims at offsetting current emissions but has no impact on the firm's production technology.[1] At the other extreme, we can think of green innovation, which aims at making the firm's production technology more sustainable but has a delayed outcome—thus, it has no impact on current emissions. An intermediate type between the two extremes is the adoption of existing green technologies—in the following, simply referred to as adoption. The impact of adoption on sustainability is less path-breaking than green innovation but, differently from green innovation, makes an immediate dent on current carbon emissions. At the same time, adoption is more expensive than abatement but, differently from abatement, its impact is not entirely transient.

As a first prediction, our analysis reveals that tighter financing frictions dampen a firm's sensitivity to its own technological sustainability—or, as we call it, the firm's marginal value of sustainability. From the shareholders' perspective, we show that the marginal benefit of making technology more sustainable is strictly greater in the case of a firm subject to no financing frictions compared to a financially constrained firm. The result holds as financing frictions decrease firm value and increase the marginal value of cash, which both make shareholders undervalue sustainability. Relatedly, in a financially constrained firm, higher costs of external funding as well as lower financial slack (i.e., lower cash reserves) decreases the marginal value of sustainability from the firm's perspective. In other words, the firm's engagement in becoming more sustainable is subordinated to the firm's financial strength. This result is consistent with the evidence in Hartzmark and Shue (2023), who show that polluting firms exhibit greater engagement in becoming greener when their financing conditions also improve.

Our analysis further shows that financing frictions have a non-monotonic effect on green

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[1] Some examples of this type of projects are afforestation and reforestation projects, or investment in international carbon offset projects.

investment as sorted by the extent of its impact on sustainability. Namely, our model shows that financing frictions have (1) no impact on abatement, (2) a strictly negative impact on adoption, and (3) an ambiguous impact on green innovation. First, abatement does not affect the firm's technological sustainability but only impacts current emissions. Because the firm chooses its abatement rate to reduce its carbon tax liability while minimizing the associated abatement costs, the resulting optimal policy is independent of the firm's financial slack. Second, adoption unambiguously declines in the presence of financing frictions. In particular, financing frictions erode the benefit of adoption—which is driven by the marginal value of sustainability—and increase its cost—because cash is more valuable in the presence of financing constraints. Third, the ambiguous effect on green innovation results from two opposing strengths. On one hand, financing frictions increase the effective cost of investing in R&D, as financial slack becomes more valuable. On the other hand, technological breakthroughs also become more valuable as they relax financing constraints, consistent with the empirical evidence (see, e.g., Farre-Mensa, Hegde, and Ljungqvist, 2020; Hall, 2019).

To the extent that the impact of financing frictions on innovation is ambiguous, we further characterize which firms should exhibit a greater engagement in green innovation. Whereas firms with stable cash flows decrease their engagement in green innovation as financing frictions tighten, we show that firms with sufficiently volatile cash flows exhibit the opposite behavior. In particular, as financial slack decreases, firms with highly volatile cash flows are relatively insensitive to the associated costs of innovation, and instead become more keen on attaining a green breakthrough that will ultimately relax financing constraints. Interestingly, as the oil and gas sector is characterized by highly volatile cash flows, our model is then capable of explaining this sector's substantial engagement in green innovation—as documented by Cohen, Gurun, and Nguyen (2025); Chiu, Hsu, Li, and Tong (2024); Kruse, Mohnen, Pope, and Sato (2020)—despite the increasing challenges in raising outside financing given the sector's high carbon emissions' profile.

Our framework allows to study both the immediate impact of financing frictions on a firm's current emissions as well as their effect on the firm's engagement to transition to more sustainable technologies—two complementary dimensions. Namely, we show that financing frictions lead to a reduction in current carbon emissions, as financially constrained firms subject to carbon taxes seek to limit their carbon tax liability by cutting on production. This is environmentally good but has a short-term impact. At the same time, financing frictions have an ambiguous impact on the speed of transition to sustainable technologies (or, more simply, the "transition rate"), stemming from the negative impact on adoption and the ambiguous impact on green innovation. Under the realistic assumption that firms put a greater emphasis on adoption than on innovation (for instance, as the former is significantly cheaper), financing frictions slow down the transition rate. This is environmentally bad, and even more so because it has a long-lasting impact. These results appear to be consistent with the available evidence, whereby financing frictions lower a firm's net emissions because they lead to a reduction in its scale of production (see Sheldon, 2017), but they also induce the firm to deprioritize green investment (see Peters, Marland, Le Quere, Boden, Canadell, and Raupach, 2012). Consistent with our model, Pacca, Antonarakis, Schroder, and Antoniades (2020) emphasize the differential short-term versus long-term effect of a tightening in financing frictions on firm's carbon footprint: While financing frictions may lead to lower current emissions in the short term due to a contraction in production, firms' engagement in building sustainable industrial processes also slows down.

Given our focus on financial constraints, our setting is particularly well-suited to study the impact of socially responsible investment on the different tools through which firms can become less polluting. Namely, we study policies such as exclusion (i.e., denying financing to polluting firms) and tilting (i.e., making financing gradually cheaper as firm take steps to becoming sustainable). Our analysis illustrates that exclusion strategies are particularly effective at reducing current carbon emissions by inducing polluting firms to reduce their production rates. Yet, exclusion strategies fail to sustain the investment in cleaner technologies and, thus, slow down firms' transition rate. Conversely, tilting strategies have the potential to ensure a faster transition to cleaner technologies, although this is mostly adoption-driven (as opposed to innovation-driven). Specifically, although tilting does not result in greater engagement in green innovation unconditionally—whereby the outcome depends on the firm's specific cash position, its cash flow volatility, and the price of carbon emissions, among other aspects—it unambiguously sustains the adoption of existing green technologies.

As an alternative way of channeling resources to firms to ease their green transition, subsidies have also been heatedly discussed. In particular, regulators have contemplated two types of subsidies: those encouraging green innovation and those incentivizing the adoption of existing sustainable technologies (as those described, for instance, by Accetturo, Barboni, Cascarano, Garcia-Appendini, and Tomasi, 2023). We find that the latter type has the potential to "kill two birds with one stone": While its main goal is easing investment in adoption, it also reduces financial constraints, boosts firm value, and increases the marginal value of sustainability from the firm's perspective. Hence, subsidies to adoption also spur investment in green innovation—a novel prediction that our model delivers.

Last, our analysis shows that carbon pricing induces firms to put more emphasis on short-term, transient measures to combat pollution—such as abatement—than on long-term measures—such as green innovation. Whereas Bustamante and Zucchi (2024a) show this in a setting with no financial constraints, our paper shows that this result continues to hold when allowing for financing frictions. Moreover, even if both financing frictions and carbon taxes impose a financial burden on polluting firms, our analysis illustrates that they have a distinct impact on the firm's optimal carbon emission management. Namely, an increase in carbon taxes leads to an increase in abatement and a drop in both adoption and green innovation. In turn, tighter financing frictions have no impact on abatement, a negative effect on adoption, and an ambiguous impact on green innovation.

Related literature Our paper relates to the literature analyzing the impact of financial constraints on firm's green commitments, pioneered by Heinkel, Kraus, and Zechner (2001). In this strand, Oehmke and Opp (2024) investigate the conditions under which socially responsible investors can achieve impact by influencing firms to tilt their production technology towards lower carbon emissions in a Holmstrom and Tirole (1997)-type of model. Heider and Inderst (2022) also use a Holmstrom and Tirole (1997)-type of model but they focus on environmental regulation in industry equilibrium. Edmans, Levit, and Schneemeier (2023) show that tilting—i.e., holding a brown stock if the firm has taken a corrective action—is a more effective strategy than exclusion of brown stocks for externalities reduction, as such strategy can encourage action of brown stocks too. Lanteri and Rampini (2023) focus on the adoption of green technologies by heterogeneous firms subject to financing constraints. They show that, in equilibrium, cleaner and newer capital requires a larger down payment and, thus, financially constrained, smaller firms optimally invest in dirtier and older capital than unconstrained larger firms. Doettling and Rola-Janicka (2023) model financing frictions as collateral constraints and focus on its interaction with physical risk. Lastly, Morse and Sastry (2024) review the literature on the role of net zero commitments of banks. Our framework adds to this literature by allowing a rich characterization of the firm's green investment policies, which enables us to study the tradeoff between lowering current emissions vis-a-vis transitioning to more sustainable technologies when firm's financial resources are scarce.

As noted by Bowen and Stern (2010), periods of heightened financing frictions can be opportunities for environmental improvements—immediate reduction in industrial output might lead countries towards more sustainable resource use. But the effect can also be negative if balancing budgets takes precedence over social or environmental aids by governments." At the firm level, this short- versus long-term impact of financial frictions on green investment at the heart of our theoretical analysis and sets our model apart from previous contributions. In fact, by allowing for different types of green investment and for flexible production rates, we are able to study how financing frictions affect current emissions as well as the the speed of transition to greener technologies.

Our paper then relates to the emerging literature in empirical corporate finance studying the drivers and impediments to firms' commitments to becoming greener. De Hass, Martin, Muuls, and Schweiger (2025) show that both financial constraints and deficient firm management can be impediments to corporate green investment. Accetturo et al. (2023) report that access to finance significantly increases firms' likelihood to undertake green investment, and more so for healthier firms and for firms who have access to government subsidies. Bartram, Hou, and Kim (2022) investigate the real effects of the introduction of the cap-and-trade program in California and show that it induced financially constrained firms to adapt operations, for instance by reallocating production. Hartzmark and Shue (2023) provide evidence suggesting that starving firms of capital fails to facilitate the green transition of highly polluting firms, a conclusion supported by the predictions of our model. Overall, our paper provides theoretical guidance for further empirical work on the topic as more data becomes available, by illustrating how financing frictions affect different types of green investment as well as other real and financial choices.

Finally, our paper is related to the strand of corporate finance models with financing frictions, in which firms have incentives to keep cash reserves and manage such cash reserves dynamically. Papers in this strand include Bolton, Chen, and Wang (2011, 2013), Décamps et al. (2011), Hugonnier, Malamud, and Morellec (2015), Malamud and Zucchi (2019), Della Seta, Morellec, and Zucchi (2020), Décamps, Gryglewicz, Morellec, and Villeneuve (2017), and Zucchi (2024), among others. Our contribution relies on adding sustainability considerations to this strand of the literature, which is particularly important to understand, at the firm level, how finance can influence the transition to a green economy.

The paper is structured as follows. Section 2 describes the model. Section 3 solves the model in the benchmark case with no financing frictions. Section 4 solves the model in the case with financing frictions. Section 5 analyzes the model results. Section 7 investigates the impact of innovation subsidies. Section 8 concludes. Technical developments are gathered in the Appendix.

2 The model

Time is continuous and uncertainty is modeled by a probability space $(\Omega, \mathcal{F}, P)$ equipped with a filtration $(\mathcal{F}_t)_{t\geq 0}$ that represents the information available at time t. The economy admits a constant risk-free rate ρ, and agents are risk neutral.

We design a dynamic model for a financially constrained firm that is intrinsically polluting and best manages its carbon emissions. Importantly, the firm operates in a jurisdiction in which carbon emissions are priced. We denote the firm's degree of technological sustainability (or "greenness") by G_t. Increasing the firm's greenness can be attained through investments of various types but is costly.

Production, Revenues, and Carbon Pricing. The firm's choice of production Y_t affects its revenues. For simplicity, we consider that the firm is a monopolist, and we normalize the cost of production to zero. We assume that the firm faces the following inverse demand function:

$$p(Y_t) = a - b\frac{Y_t}{G_t} \quad a > 0\,, b > 0. \tag{1}$$

In this expression, the sensitivity of prices to Y_t is scaled by the firm's sustainability G_t to capture that consumers are sensitive to the firm's environmental engagement. Thus, the more sustainable the firm is, the greater the amount of product demanded by consumers for a given price level. This assumption is consistent with the growing evidence that consumers reward greener firms (see, e.g., Derrien et al. (2023), Meier et al. (2023)).[2] More broadly, this demand function is consistent with the argument that less sustainable inputs entail greater cost of production as in Acemoglu, Aghion, Bursztyn, and Hemous (2012), which in turn delivers pro-sustainability preferences as shown by Bustamante and Zucchi (2024a).

The firm's production process also generates carbon emissions and, thus, carbon related

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[2] Derrien et al. (2023) show that analysts significantly downgrade earnings forecasts on a firm following negative ESG news on such firm. They show that the negative revision of earnings forecasts reflects expectation of lower sales rather than higher future costs. Using barcode-level sales data, Meier et al. (2023) show that environmental and social ratings positively relate to firm sales.

expenses. We denote the firm's gross emissions by νY_t per unit of time, where the parameter ν denotes the firm's emission intensity and should be thought of as an industry-specific parameter. Absent any type of green investment, the firm pays a cost κ per tonne of (gross) carbon emissions. Nonetheless, as we describe next, the firm can also decrease its carbon tax liability by engaging in green investments of various types. Hence, whereas a firm's carbon footprint has an industry-specific component determined by ν, it is also shaped by the firm-specific green investments.

Sustainability and green investment The model allows for three different types of green investment: Abatement projects, adoption of (existing) greener technologies (henceforth, adoption), and green innovation.

First, abatement projects immediately offset carbon emissions but do not change the firm's technology (see, e.g., Kim, Li, and Wu, 2024). That is, these projects simply aim at cleaning up some of the firm's emissions ex-post. Examples of abatement are afforestation or reforestation projects as well as contributions to international carbon offset projects. Abatement leads to a reduction $s_t G_t$ in the firm's current carbon emissions and entails a quadratic cost $\frac{\alpha}{2} s_t^2 G_t$.[3] Notably, the effect of abatement projects is transient because it only reduces emissions in the present but does not impact the firm's technological sustainability G_t. In other words, if the firm wants to reduce emissions by a given amount in each period through abatement projects, it has to undertake the associated expenditure in each period.

Second, the firm can improve the sustainability of its production processes by adopting existing green technologies. In the following, we refer to this type of investment as adoption. Some examples of adoption are installing solar panels, investing in recycling processes, or switching to electric vehicles. Adopting greener technologies leads to an increase in the firm's stock of technological sustainability. Namely, the firm's technological sustainability increases at rate i_t if the firm spends the cost $\frac{\theta}{2} i^2 G_t$.

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[3] For tractability, we assume that both the benefit and cost of abatement scale with G_t. This is without loss of generality as, because both the benefit and cost scale up with G_t, we can think of the optimal abatement policy as independent of G_t.

Third, the firm can invest in green innovation—i.e., it can spend on green R&D to come up with novel, sustainable technologies. Green innovation leads to a technological breakthrough at a Poisson rate ϕz_t if the firm spends the quadratic cost $\frac{\zeta}{2} z_t^2 G_t$. Innovation becomes more costly if the firm is already more sustainable (i.e., G_t is higher).[4] This formulation also implies that the more the firm invests in green innovation, the higher the rate of arrival of a green breakthrough. When such breakthroughs happen, the firm's sustainability G_t increases lumpily by a factor $\lambda > 1$.

Under these assumptions, the firm's sustainability (or greenness) exhibits the following dynamics:

$$dG_t = (i_{t-} - \delta) G_{t-} dt + (\lambda - 1) G_{t-} dN_t \tag{2}$$

where δ captures the depreciation rate of firm's technological sustainability. The first term implies that, on each time interval, the firm's sustainability grows incrementally due to the adoption of available greener technologies. The second term shows that the firm's sustainability can also increase lumpily thanks to green technological breakthroughs, where N_t is a Poisson process with endogenous intensity ϕz_t.

Notably, abatement does not affect the firm's sustainability (as indeed it does not enter the dynamics of equation (2)) but it immediately affects the firm's *net* emissions, which are given by:[5]

$$\nu Y_t - s_t G_t - \xi G_t \tag{3}$$

That is, the greater the firm's investment in abatement, the lower the firm's current carbon emissions. Moreover, the impact of abatement projects on net emissions is transient, as the firm has to undertake abatement expenditures every period to indeed offset a given volume of its carbon emissions. In turn, an increase in G_t leads to a permanent decline in the firm's emissions (G_t is a stock variable that accumulates over time). Namely, the greater the term $\xi > 0$, the more G_t leads to a decline in sustainability—thus, investment

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[4] This assumption is shared with previous models of innovation and growth, see e.g., Akcigit and Kerr (2018) or Acemoglu and Cao (2015).

[5] For the sake of simplicity, we assume that if the firm's net emissions are negative—i.e., the firm is a net cleanser—it then receives a subsidy. In our analysis, however, we mostly focus on polluting firms.

in adoption as well as green innovation breakthroughs permanently improve the firm's technological sustainability and decrease emissions for a given scale of production. This modeling is similar to Bustamante and Zucchi (2024a).

Importantly, the different types of green investment can be sorted by their impact on sustainability. At one extreme, abatement has no impact at all on sustainability and, thus, on the firm's production technology—namely, abatement immediately decreases carbon emissions but has no effect on G_t. Second, adoption leads to an incremental improvement in the firm's technological sustainability, which has a direct impact on the firm's carbon emissions. Lastly, green innovation expenditures have a delayed but lumpy (path-breaking) impact on technological sustainability—thus, it does not impact carbon emissions today, but has a long-lasting effect on the sustainability of the firm's production processes.

Cash flows, financing frictions, and cash holdings. Under the assumptions described above, the firm's operating profits satisfy the following dynamics:

$$dX_t = \left[p(Y_t)Y_t - \kappa(Y_t\nu - s_tG_t - \xi G_t) - \left(\frac{\alpha s_t^2}{2} + \frac{\theta i_t^2}{2} + \frac{\zeta z_t^2}{2} \right) G_t \right] dt + \sigma Y_t dB_t, \qquad (4)$$

The first term suggests that profits increase with sales (the first term in square brackets) whereas decrease with the carbon tax (the second term) as well as with abatement, adoption, and green innovation expenditures (the last term in square brackets). Nonetheless, abatement reduces the firm's carbon tax burden, as the firm gets taxed on net carbon emissions, defined in (3). The last term captures cash flow volatility. Namely, the greater the firm's scale of production, the greater the volatility of its cash flows.[6]

The specification in equation (4) implies that the firm can make operating profits or losses on any time interval. Such losses could be covered through external financing if it was costless. We relax this assumption and acknowledge that firms face costs when raising external funds. Following previous works (BCW, 2011, 2013; Décamps et al., 2011; Hennessy and Whited, 2007), we model financing frictions in a reduced-form fashion to

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[6] This modeling is also adopted in Bustamante and Zucchi (2024b) or Malamud and Zucchi (2019).

capture asymmetric information problems. Namely, we assume that whenever the firm raises outside financing, it bears a proportional cost χ and a fixed cost, F_t. We assume that the fixed cost scaled by the firm's sustainability stays constant over time and denoted by f, to avoid that the firm ever outgrows its cost of financing.[7] In Section 6.2, we relax this assumption and see how policies would change if the firm outgrows its fixed cost if it becomes sufficiently sustainable—consistent with observed policies of tilting and, thus, cheaper financing to greener firms.[8]

The empirical literature highlights how successful innovation results in enhanced availability of external financing (see, e.g., Farre-Mensa, Hegde, and Ljungqvist (2020)).[9] Whereas these previous works focus on innovation in general, this should be even more the case in the *green* innovation field, where such innovations are craved by both investors and regulatory bodies. Indeed, Hege, Pouget, and Zhang (2023) report that climate-related patents ease the financing conditions of the awarded firm. Similar to the modelling approach in Clementi and Hopenhayn (2006) and Malamud and Zucchi (2019), we thus acknowledge that financing frictions loosen up when a firm attains an innovation breakthrough. To capture this idea, we assume that the firm is waived the fix and variable financing costs upon a green innovation breakthrough.[10]

The presence of financing frictions implies that the firm has incentives to accumulate cash. However, holding cash entails an opportunity cost, as the return on cash—denoted by r—is strictly lower than the risk free rate, i.e., $r < \rho$. As in previous cash management models, the opportunity cost of cash can be motivated by a tax disadvantage (Graham, 2000) or a free cash flow problem (Jensen, 1986). The dynamic of the firm's cash reserves,

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[7] This assumption is equivalent to assuming the absence of socially-responsible investors—who make financing cheaper to greener firms—and serves as a useful benchmark.

[8] In other words, our baseline setup does not embed tilting, though we gauge its effect separately in Section 6.2.

[9] Hall (2019) provides a survey and discusses alternative channels through which successful innovation relaxes financial constraints. Importantly, such achievement likely alleviates the asymmetric information problem between the firm and outside investors.

[10] We assume that financing costs go to zero in this case just for simplicity, but one could think of a model in which routine financing is relatively more expensive than financing upon a breakthrough.

denoted by W_t, are described by the following equation:

$$dW_t = rW_t dt + dX_t + dO_t - d\Phi_t - dU_t \tag{5}$$

where O_t is the cumulative process representing outside financing, Φ_t is the cumulative issuance costs, and U_t represents cumulative dividends. This equation implies that cash reserves increase with the return on cash, with operating profits, and with outside financing, whereas they decrease with operating losses, financing costs, and dividend payments.

As in previous cash management models, the firm should maintain a non-negative cash reserve as an operating constraint. If the firm depletes its cash reserves and does not raise outside financing then, it has to liquidate its assets. We denote the firm liquidation value by L. Furthermore, we assume that the firms' liquidation value is increasing in its sustainability, so that $L = \ell G_t$. As the firm becomes more sustainable, it adopts valuable green technologies, which ultimately allow a higher recovery rate if liquidated.

Optimization Management chooses the firm's production (Y), abatement (s), adoption (i), green innovation (z), payout (U), financing (O), and default (τ) policies to maximize shareholder value. That is, management solves:

$$\sup_{(Y,s,i,z,U,O,\tau)} \mathbb{E}_{w,i} \left[\int_0^\tau e^{-rt} \left(dU_t - dO_t \right) + e^{-r\tau} \max \left\{ 0; \ell + W_\tau \right\} \right] \tag{6}$$

subject to equation (5). The first term on the right-hand side of equation (6) represents the flow of dividends accruing to incumbent shareholders, net of the claim of new shareholders. The second term represents the present value of the cash flow to shareholders in default, which is given by the liquidation value of assets plus the firm's cash reserves at that time.

3 Benchmark case with no financing frictions

We start by solving the model in the benchmark case with no financing frictions. In this case, the firm has no incentives to keep cash as it can raise external financing when needed at no cost. Financing and dividend policies are trivial. Moreover, as long as it is expected to be profitable, the firm never liquidates its assets.

In this setup, firm value is solely a function of its sustainability G_t. The firm has incentives to become more sustainable to decrease its carbon tax liability as well as to increase its product demand (see equation (1)). In this setting, the dynamics of firm value satisfy the following HJB equation:

$$\rho V^*(G) = \max_{Y^*,s^*,i^*,z^*} \left[p(Y^*)Y^* - \frac{(s^*)^2\alpha}{2}G_t - \frac{(z^*)^2\zeta}{2}G_t - \frac{(i^*)^2\theta}{2}G_t - \kappa\left(Y^*\nu - s^*G - \xi G\right)\right] + (i^* - \delta)GV_G^* + \phi z^*\left[V^*(\lambda G) - V^*(G)\right].$$

The left-hand side of this equation is the return required by shareholders. The right-hand side is the expected change in firm value on each time interval. Specifically, the first term on the right-hand side captures the firm's profits, hereby given by sales net of carbon taxes and net of its investment in abatement, in adoption, and in green innovation. The second term represents the effect of infinitesimal changes in the firm's degree of sustainability. By contrast, the third term considers the impact of discrete changes in sustainability coming from green innovation breakthroughs.

To solve for the firm's problem, we conjecture that $V^*(G) = Gv^*$ and $Y_t^* = Gy^*$, where v^* (respectively, y^*) denotes firm value (production) scaled by sustainability. Differentiating the resulting scaled HJB equation (see equation (A.1) in Appendix A.1) with respect to production gives:

$$y^* = \frac{a - \nu\kappa}{2b}. \tag{7}$$

If there was no carbon pricing (meaning that $\kappa = 0$) the firm's optimal scale of production would be $\frac{a}{2b}$. Thus, the equation suggests that carbon pricing leads firms to reduce their optimal scale of production: The greater the cost of carbon κ, the lower the firm's production (see also Bustamante and Zucchi (2024a)). Also, the more polluting the industry in which the firm operates—captured by a greater ν—the lower the firm's scale of production.

Next, consider the firm's optimal engagement in abatement. Differentiating the scaled HJB equation gives

$$s^* = \frac{\kappa}{\alpha}. \tag{8}$$

This equation suggests that the larger the cost of carbon κ, the greater the firm's investment in abatement, consistent with Bustamante and Zucchi (2024a). In turn, the higher the cost of abatement α, the lower the firm's optimal abatement rate.

The firm also decides how much to invest in the adoption of existing green technologies. Differentiating the scaled HJB equation with respect to i^* gives

$$i^* = \frac{v^*}{\theta}. \tag{9}$$

The larger v^* is, the more the firm invests in the adoption of existing greener technologies. In turn, if the cost of green adoption is higher, then the firm invests less in adoption.

Lastly, differentiating the scaled HJB equation with respect to z^* gives the firm's optimal investment in green innovation

$$z^* = \frac{\phi(\lambda - 1)v^*}{\zeta}. \tag{10}$$

This expression illustrates that the greater the likelihood of a green breakthrough or the more significant its surplus, the greater the firm's engagement in green innovation. On the contrary, the equation suggests that the higher the cost of innovation ζ, the lower the firm's incentive to invest in innovation.

Substituting the firm's optimal policies into the HJB equation gives an explicit solution for firm value absent financing frictions

$$v^* = \frac{(\delta+\rho) - \sqrt{(\delta+\rho)^2 - 2\left(\frac{1}{\theta} + \frac{\phi^2(\lambda-1)^2}{\zeta}\right)\left(\frac{(a-\nu\kappa)^2}{4b} + \frac{\kappa^2}{2\alpha} + \xi\kappa\right)}}{\left(\frac{1}{\theta} + \frac{\phi^2(\lambda-1)^2}{\zeta}\right)}. \tag{11}$$

As shown in Appendix A.1, firm value decreases with κ as long as the firm exhibits positive emissions. Thus, polluting firms have incentives to reduce emissions to increase shareholder value when subject to carbon pricing.

4 Model solution with financing frictions

Financing frictions lead the firm to accumulate cash reserves. As in previous cash management models, the benefit of holding cash decreases with cash reserves. In turn, the opportunity cost of holding cash is constant and equal to $\rho - r$. We thus conjecture that there is a target cash level W^* at which the costs and benefit of cash are equalized. Above W^*, it is optimal to pay excess cash as dividends to shareholders. Below W^*, shareholders retain earnings in cash reserves and search for financing. The cash reserves of the firm always remain nonnegative as an operating constraint—if out of cash, the firm would not be able to cover any operating losses.

Differently from the case with no financing frictions, the value of the firm is not only a function of its sustainability, but also of its cash reserves. Firm value is then denoted by $V(G, W)$. For any $W < W^*$, firm value satisfies:

$$\begin{aligned}\rho V(G,W) = \max_{Y,s,i,z}\ & \left[rW + p(Y)Y - \frac{s^2\alpha}{2}G_t - \frac{z^2\zeta}{2}G_t - \frac{i^2\theta}{2}G_t - \kappa(\nu y - s - \xi)G\right]V_W \\ & + \frac{\sigma^2}{2}Y^2 V_{WW} + (i-\delta)GV_G + \phi z\left[V(\lambda G, W^*(\lambda G)) - V(G,W) - (W^*(\lambda G) - W)\right]. \end{aligned} \tag{12}$$

The left-hand side represents the return required by shareholders. The right-hand side is the expected change in firm value on each time interval. The first term represents the effect of infinitesimal changes in cash reserves on firm value. Cash reserves increase with the return on cash and with sales, whereas they decrease with the carbon tax and the firm's investment in abatement, adoption, and green innovation. The second term captures the impact of cash flow volatility. The third term represents the impact of an infinitesimal change in the firm's degree of sustainability, propelled by the firm's continuous investment in adoption. The fourth term captures the impact of a green innovation breakthrough, in which case the firm's sustainability increases discretely, and the firm takes advantage of the associated relaxation of financing frictions by replenishing cash reserves up to the target level.

To solve the model, we conjecture that firm value satisfies:

$$V(G,W) = Gv\left(\frac{W}{G}\right) = G_t v(w_t) \quad w_t \equiv \frac{W_t}{G_t} \tag{13}$$

so scaled cash becomes our state variable.[11] We also define scaled production as $y_t \equiv \frac{Y_t}{G_t}$. Substituting into the above HJB and differentiating it with respect to y gives the optimal scale of production:

$$y = \frac{(a - \kappa\nu)v'}{2bv' - \sigma^2 v''}. \tag{14}$$

Notably, the denominator implies that the more concave the value function is (i.e., the more the firm is effectively risk averse), the lower the firm's production rate. Moreover, the higher the cost of carbon κ, the lower the firm's production rate, and more so if the marginal value of cash v' is higher.

Differentiating the resulting scaled HJB equation (i.e., equation (33)) with respect to s gives the firm's optimal abatement policy:

$$s = \frac{\kappa}{\alpha}. \tag{15}$$

As in the case with no financing frictions, the greater the cost of emissions, the more the

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[11] Unscaled cash (and target) is capitalized, whereas scaled is not.

firm invests in abatement. Because abatement does not change the firm's technology but only affects the firm's cash flows—balancing the abatement costs vis-a-vis the ensuing lower carbon tax liability stemming from lower net emissions—the optimal abatement policy does not depend on the firm's cash holdings.

Differentiating with respect to i gives instead the optimal adoption rate:

$$i = \frac{1}{\theta v'} \left[v - w v' \right] \tag{16}$$

Differently from the benchmark case with no financing frictions, cash and its marginal value affect the firm's investment in adoption.[12] Namely, the greater the marginal value of cash v', the lower the firm's investment in adoption. In turn, if adoption is costlier (as embodied by θ), the lower the firm's engagement in adoption, and more so if the marginal value of cash v' is greater.

Finally, differentiating the scaled HJB equation with respect to z gives the optimal green innovation rate:

$$z = \frac{\phi}{\zeta v'} \left[\lambda v(w^*) - v(w) - (\lambda w^* - w) \right]. \tag{17}$$

I.e., the greater the surplus from innovation (i.e., the numerator), the greater the firm's innovation rate. In turn, if green innovation is costlier (as embodied by ζ), the lower the firm's engagement in green innovation, and more so if the marginal value of cash v' is greater.

To solve for firm value, we substitute (14), (15), (16), and (17) into (33) and solve the resulting equation subject to the following boundary conditions. First, if the cash reserves exceeds its target level w^*, the firm pays out cash to shareholders. Firm value is thus linear for any $w \geq w^*$: $v(w) = v(w^*) + w - w^*$. Subtracting $v(w)$ from both sides of this equation,

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[12] As in the case without financing frictions, in Section 5 we illustrates that the numerator is the marginal value of sustainability, i.e., $V_G = v - wv'$ (see the Appendix for calculations). The greater the marginal value of sustainability, the larger the firm's engagement in adoption.

dividing by $w - w^*$, and taking the limit $w \to w^*$ gives:

$$v'(w^*) = 1. \tag{18}$$

That is, it is optimal to start paying out cash when the marginal value of one dollar inside the firm equals the value of a dollar paid out to shareholders. The target cash level that maximizes shareholder value is determined by the super-contact condition:

$$v''(w^*) = 0. \tag{19}$$

At $w = 0$, the firm raises new funds, so the following condition holds

$$v(0) = v(w_*) - (1 + \chi)w_* - f, \tag{20}$$

where w_* denoted the optimal issuance size pinned down by the following condition

$$v'(w_*) = 1 + \chi. \tag{21}$$

This equation implies that the marginal gain from raising cash (the left-hand side) equalizes its marginal cost (the right-hand side). Notably, the firm is better off raising external financing than liquidating its asset at $w = 0$ if the following inequality holds:

$$v(w_*) - (1 + \chi)w_* - f > \ell \tag{22}$$

i.e., if the continuation value from raising funds (the left-hand side) is greater than the liquidation value of the assets (the right-hand side). When equation (22) does not hold, in turn, the firm finds it optimal to liquidate instead of refinancing. In this case, the boundary condition at $w = 0$ is

$$v(0) = \ell, \tag{23}$$

meaning that, at $w = 0$, firm value equals its liquidation value.

5 Model analysis

5.1 Financing frictions, sustainability, and firm policies

Comparing optimal policies against the frictionless benchmark In our model embedding both financial and environmental considerations, firm value is driven by two state variables: The firm's cash reserves (W) and the firm's degree of sustainability (G). First, cash reserves serve as a buffer that relaxes the firm's financial constraints. Similar to previous cash management models, the firm's sensitivity to cash—i.e., the marginal value of cash $V_W = v'$—is an important driver of firm's optimal choices and decreases as cash reserves increase.[13] Second, the firm's sensitivity to sustainability is given by V_G, representing how much firm value varies in response to a change in G. Henceforth, we will denote this quantity as the marginal value of sustainability. We have the following result.

Lemma 1 *The marginal value of sustainability is* $V_G^* = v^*$ *absent financing frictions and* $V_G = v(w) - wv'(w)$ *in the presence of financing frictions. As long as financing frictions are value decreasing, they make the firm less sensitive to sustainability, i.e.* $V_G^* > V_G$.

Absent financing frictions, the marginal value of sustainability is given by scaled firm value. Conversely, in the presence of financing frictions, the marginal value of sustainability equals scaled firm value net of cash weighted by its marginal value. Because financing frictions both reduce shareholder value and increase the marginal value of cash above one, the marginal value of sustainability is then lower for financially constrained firms vis-a-vis financially unconstrained firms. Importantly, financing frictions lead the firm to underweigh the value of becoming technologically sustainable compared to a benchmark case with no frictions. The more financially constrained the firm is, the lower its sensitivity to

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[13] That is, as the marginal value of cash decreases with w, $v'' < 0$—i.e., the firm is effectively risk averse, a standard result in cash management models.

sustainability. It follows that sustainability is subordinated to the firm's financial health, which is consistent with the prediction in Hartzmark and Shue (2023) that polluting firms with high emission intensity are more engaged to becoming greener when their financing conditions ease.

Comparing the optimal policies in the presence and in the absence of financing frictions, we have the following result.

Lemma 2 *In the presence of financing frictions:*

1. *The production rate never exceeds that in the benchmark with no frictions, i.e.,* $y(w) \leq y^*$*, as long as financing frictions make the firm effectively risk averse;*
2. *The optimal abatement rate is the same as in the benchmark case with no frictions, i.e.,* $s(w) = s^*$ *for any* $w < w^*$*;*
3. *Investment in adoption is lower than in the benchmark with no frictions, i.e.,* $i(w) < i^*$ *for any* $w \leq w^*$ *as financing frictions reduce the marginal value of sustainability;*
4. *At* $w = w^*$*, financing frictions have a negative impact on innovation, i.e.,* $z(w^*) < z^*$*. Yet, the effect is ambiguous for* $w < w^*$*.*

Lemma 2 compares firm policies in the presence and in the absence of financing frictions. First, as long as financing frictions make the firm effectively risk averse (as in previous dynamic cash management models, $v'' < 0$), the presence of financing frictions leads the firm to produce less in order to limit its cash flow volatility, and more so as w gets closer to zero. This implies that financing frictions lead to a reduction in a firm's gross carbon emissions νy, which are the lowest as w approaches zero.

Second, financing frictions have a non-monotonic effect on green investment when sorted by its impact on sustainability: no impact on abatement (claim 2), a negative impact on adoption (claim 3), and an ambiguous impact on green innovation (claim 4). Consider abatement first. As a distinguishing feature, abatement does not change the firm's technology but simply impacts the firm's current emissions. Whereas abatement is costly—thus eroding the firm's cash flows—it directly and immediately reduces the firm's carbon tax liability—thus increasing the firm's cash flows. The optimal abatement rate then balances these two strengths and is independent of the firm's cash reserves.

In turn, financing frictions have a negative impact on the optimal investment in adoption. This effect is driven by two strengths. First, financing frictions erode the marginal value of sustainability as illustrated by Lemma 2 (the numerator in the expression of $i(w)$)—this strength leads to a decrease in the benefit from adoption and should push the adoption rate down. Second, financial constraints impact the effective cost of adoption (the denominator in the expression of $i(w)$), as θ is weighted by the marginal value of cash. Because the marginal value of cash is above one in the presence of financing frictions, the effective cost of adoption increases compared to the case with no frictions—a strength that should also push the adoption rate down. Overall, financial frictions unambiguously slow down the firm's adoption of green technologies, consistent with many studies, including Accetturo et al. (2023) and De Hass et al. (2025).[14]

Lastly, Lemma 2 shows that financing frictions have a negative impact on the firm's innovation rate at $w = w^*$. Nonetheless, financing frictions have an ambiguous effect on z for $w < w^*$. The reason is the following. On the one hand, financing frictions raise the cost of investing in green innovation (i.e., the denominator in equation (17)) as ζ is weighted by the marginal value of cash, which again is higher than one in the presence of frictions. On the other hand, however, the benefit from innovation increases in the presence of financing frictions: Indeed, once attained, technological breakthroughs enable the firm to access cheaper financing. This benefit is greater if the firm holds smaller cash reserves. Proposition 4 and our numerical analysis (see Section 5.2) shows that both strengths can dominate, and clarifies for which firm types the latter strength prevails.

Sensitivity of optimal policies to financial slack So far, we have investigated how financing frictions affect corporate policies by comparing the case with financing frictions

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[14] This result also resonates Kaldorf and Shi (2024) who points to financial barriers in the adoption of green technologies.

against the benchmark environment with no frictions. Next, we focus on another important dimension: How cash reserves—i.e., the firm's financial slack—affect the firm's sensitivity to sustainability and then optimal policies. We have the following result.

Lemma 3 *If financing frictions make the firm effectively risk averse ($v'' < 0$ for any $w < w^*$), then:*

- *The firm's sensitivity to sustainability V_G increases with cash reserves w;*
- *The optimal abatement rate s is insensitive to cash reserves w;*
- *The optimal adoption rate i increases with cash reserves w;*
- *The optimal green innovation rate z exhibits an ambiguous sensitivity to cash reserves.*

Lemma 3 shows that the firm becomes less sensitive to sustainability as its cash reserves decrease. The reason is that the marginal value of cash increases as w decreases, then eroding the marginal value of sustainability. In other words, a larger stock of cash makes the firm more financially flexible (equivalently, less financially constrained), which in turn increases the marginal value of sustainability. Therefore, financial slack increases the firm's sensitivity to sustainability and, thus, its optimal investment in adoption.

In turn, cash reserves have no impact on abatement (as discussed in the context of Lemma 2) and have an ambiguous effect on green innovation. On the one hand, because a technological breakthrough can relax the firm's financial constraints, the benefit from innovation decreases with the level of cash reserves—the reason being that a successful breakthrough allows the firm to access cheap financing, which is more beneficial when the cash reserves is low. At the same time, the marginal cost of investing in innovation (i.e., the denominator of $z(c)$ in equation (17)) also decreases, as cash is less valuable as cash reserves grow larger. Thus, the optimal innovation rate can both increase or decrease with cash reserves.[15] We investigate which firm characteristics trigger one pattern over the other in the next proposition.

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[15] This stems from the fact that the marginal value of cash v' should be greater than 1 and firm value should be concave for any $w < w^*$.

Proposition 4 *The optimal innovation rate z increases with cash reserves if σ is sufficiently small . When this is the case, $z(w) < z^*$ for any $w \leq w^*$.*

Proposition 4 suggests that firms with low cash flow volatility find it optimal to gradually cut their green innovation expenditures as cash reserves decrease. When cash flow volatility is low, a small reduction in cash reserves leads to a notable increase in the probability of costly refinancing (i.e., hitting the financing boundary $w = 0$).[16] In these cases, the marginal value of cash v' is highly sensitive to w when w approaches zero, and so is the effective cost of innovation (i.e., the denominator in equation (17)). Hence, when cash flow volatility is low and cash reserves decrease, the firm becomes relatively more cautious in safeguarding its financial slack and invests in innovation at a higher rate as long as its cash reserves increase. Technically, the cost of innovation (the denominator in equation (17)) increases at a higher rate than the gain (the numerator in equation (17)), so that $z(w)$ increases with w. By contrast, firms with sufficiently volatile cash flows can increase their engagement in green innovation as cash reserves fall. When volatility is high, cash reserves can be replenished or wiped out quickly by cash flow shocks. As a result, when cash holdings are close to zero, the firm becomes relatively insensitive to reducing the costs of innovation, as any saving made in terms of R&D costs can be easily wiped out by cash flow shocks. Instead, the firm becomes relatively more keen on attaining a green breakthrough in an effort to ultimately relax its financing constraints. Technically, the effective cost of innovation (the denominator in equation (17)) increases at a lower pace than the gain as w decreases (the numerator in equation (17)), so that $z(w)$ decreases with w. Notably, in this case, $z(w)$ can exceed z^* for some $w < w^*$.

Interestingly, fossil energy, oil, and gas firms have typically very volatile cash flows (see, e.g., Bugshan, 2022). Because these firms belong to highly polluting industries, they are increasingly excluded from the portfolio of socially-responsible investors—thus, they are becoming increasingly financially constrained. Nonetheless, and consistent with the predictions of our model, recent evidence suggests that these firms are largely committed to

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[16] Alternatively, firms can liquidate if optimal, i.e., if condition 22 does not hold.

green innovation. In particular, Cohen, Gurun, and Nguyen (2025) report that energy firms are key innovators in the US green patent landscape, producing more and of significantly higher quality innovations (see also Chiu et al., 2024; Kruse et al., 2020). In section 5.2, we investigate this pattern further.

Impact of optimal policies on emissions and sustainability An important question is how the optimal corporate policies described so far affect the firm's efforts to becoming greener. That is, when investigating a firm's engagement to combat climate change, there are two dimensions to consider. The first is how the firm controls current emissions by adjusting its production as well as its abatement rates. The second is how much the firm is committed to improve the sustainability of its technology, then making its emissions permanently lower for a given level of production (and irrespective of its engagement in abatement). To investigate the second dimension, we define the firm's expected increase in sustainability G as

$$\Gamma(w) = \phi z(w)\,[\lambda - 1] + i(w) - \delta. \tag{24}$$

In the following, we refer to this quantity as the the green transition rate (or speed of transition). This equation illustrates that a change in the firm's sustainability G comes from: (1) green innovation breakthroughs (the first term); (2) the adoption of available green technologies (the second term); (3) depreciation (the last term). The next result stems as an implication of the firm's optimal policies (Lemma 2 and Lemma 3).

Corollary 5 *Financing frictions lead to: (1) a reduction in the firm's emissions, $\nu y(w^*) - s(w^*) - \xi < \nu y^* - s^* - \xi$; (2) a reduction in the green transition rate $\Gamma(w)$ at least at $w = w^*$. If $z(w)$ increases with w, then $\Gamma(w) < \Gamma^*$ for any $w \leq w^*$. However, if $z(w) > z^*$ for some $w < w^*$, then $\Gamma(w)$ can exceed Γ^* for some $w < w^*$.*

The above corollary illustrates that financing frictions have both a good or a bad environmental impact depending on whether we look at current emissions or the speed of transition to greener technologies. Because financing frictions lower the firm's optimal production rate and leave abatement unchanged, a financially constrained firm exhibits lower carbon emissions compared to an identical unconstrained firm. This is environmentally good. At the same time, Corollary 5 also suggests that financing frictions have a negative impact on the firm's engagement in becoming more sustainable at least at $w = w^*$ (i.e., when the firm has its optimal level of cash reserves). In fact, at $w = w^*$, the firm invests less in adoption *and* in green innovation compared to the unconstrained benchmark, which translates into an unambiguously negative impact on the transition rate $\Gamma(w)$. This is environmentally bad. Nonetheless, if z is higher than z^* for some $w < w^*$, then the impact of financing frictions on the transition rate $\Gamma(w)$ can be ambiguous. In the following, we further investigate these patterns as well as provide more implications by numerically implementing the model.

5.2 Quantitative analysis

Baseline parameterization We start by describing the baseline parameterization used throughout the analysis. We assume that the risk free rate is equal to 0.055 and the opportunity cost of cash is 1% as in Bolton, Chen, and Wang (2011) and Décamps et al. (2011). We assume that the emission intensity ν is equal to 0.3, to match the average emissions around the world per purchasing power parity of GDP as reported by the World Bank. We assume that the carbon tax κ is equal to 10%, so that the firm's expected net emission flow is positive under our baseline parameterization.[17] We assume that $\sigma = 0.45$ which implies that cash flow volatility is equal to 12% at the target cash level, which is consistent with the values reported by Graham, Leary, and Roberts (2015). Moreover, we set $a = 0.1$ and $b = 0.13$, which gives a cash flow to assets that lies in the range of values reported by Opler, Pinkowitz, Stulz, and Williamson (1999). We set the liquidation value to 0.1, which implies that the enterprise value recovered in liquidation is consistent with the range reported by Glover (2016). We set the variable issuance cost to 5% in our baseline, which is consistent with the values reported by Hennessy and Whited (2007), and the fixed

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[17] We acknowledge that the price of carbon varies widely across jurisdictions, as well as within jurisdictions over time. As a result, we assess the robustness of our results extensively through comparative statics.

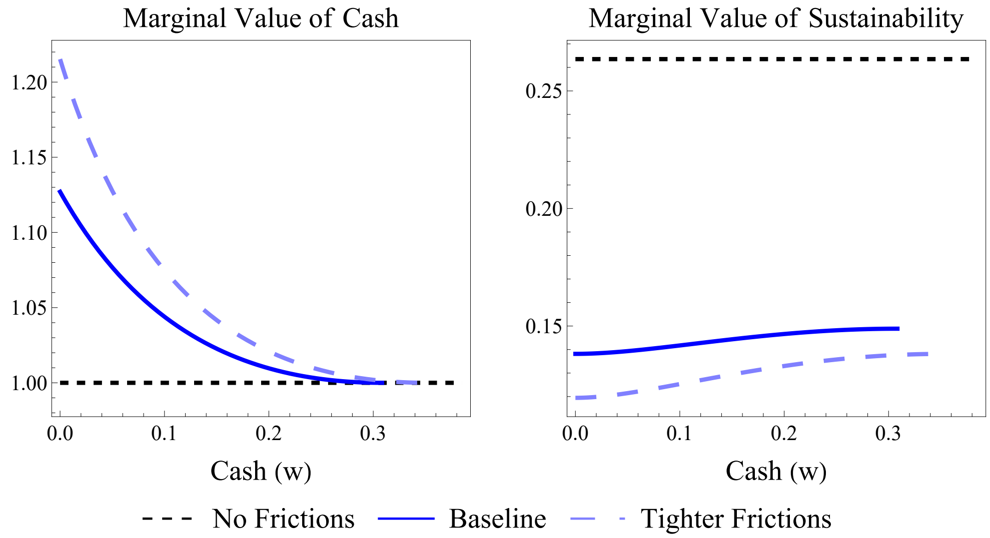

Figure 1: Marginal value of cash and marginal value of sustainability. The left panel illustrates the marginal value of cash as a function of cash reserves w. The right panel illustrates the marginal value of sustainability, again as a function of w. We consider the frictionless case with no financing frictions (black dashed line) against the financially constrained case when assuming relatively lower (solid blue line) and higher (light-blue dashed) financing costs.

cost is equal to 0.003 similar to Décamps et al. (2017). We acknowledge that abatement is cheaper than adoption, which in turn is cheaper than innovation—namely, we assume that the cost coefficient for abatement is $\alpha = 3$, for adoption is $\theta = 5$, and for innovation is $\zeta = 10$. Moreover, we assume that sustainability depreciates at rate $\delta = 0.015$. We assume that λ is equal to 1.1, which is higher than the innovation step size estimated by Acemoglu, Akcigit, Hanley, and Kerr (2016) as they allow for a breakthrough to lead to multiple steps advancements. In turn, we set $\phi = 3$ to attain an R&D expenditures-to-sales ratio that is consistent with (actually, lower than) those of Compustat firms, to account for the fact that green innovation is just a component of R&D in reality. As Bustamante and Zucchi (2024a), we set the reduction in emissions thanks to technological breakthroughs ξ equal to 0.001.

Financing frictions and firm's engagement in combating pollution Figure 1 shows the marginal value of cash and the marginal value of sustainability in the absence and in the presence of financing frictions. For the case with financing frictions, we consider two parameterizations, respectively characterized by lower (solid blue line, in which $\chi = 5\%$) or higher financing cost (large-dashed light-blue line, in which $\chi = 11\%$). As documented by Hennessy and Whited (2007), financing costs are heterogeneous across firms.

As in previous cash management models, the left panel of Figure 1 shows that the marginal value of cash decreases with w and shifts up if financing costs are larger. The higher marginal value of cash associated with tighter financing frictions erodes the marginal value of sustainability, as illustrated in the right panel. Indeed, the marginal value of sustainability is higher in the benchmark case with no frictions, and the lowest when financing frictions are the largest. This analysis then confirms that the firm's financial health precedes sustainability improvements, as concluded in Section 5.1.

Figure 2 shows the firm's optimal policies—production, abatement, adoption, and green innovation—in the same cases considered in Figure 1. Confirming our analytical results (see Section 5.1), the top left panel shows that the optimal size of production increases with cash reserves. Because effective risk aversion decreases as cash reserves increase, the firm increases its optimal scale of production, which then coincides with the unconstrained benchmark at $w = w^*$ (at which effective risk aversion is zero).

Visually illustrating Proposition 2, the top right chart of Figure 2 highlights that financing frictions have no impact on abatement. Abatement only serves to limit the cost of carbon regulation but does not change the firm technology—i.e., it only affects the firm's cash flows. That is, a higher abatement rate leads to an immediate decrease in the carbon tax, but also generates higher abatement costs. The optimal balance between the two strengths is therefore independent of the level of cash reserves as well as of the severity of financial constraints.

In turn, the bottom left chart shows that financing frictions uniformly lead to lower investment in adoption compared to the frictionless case, then quantitatively illustrating

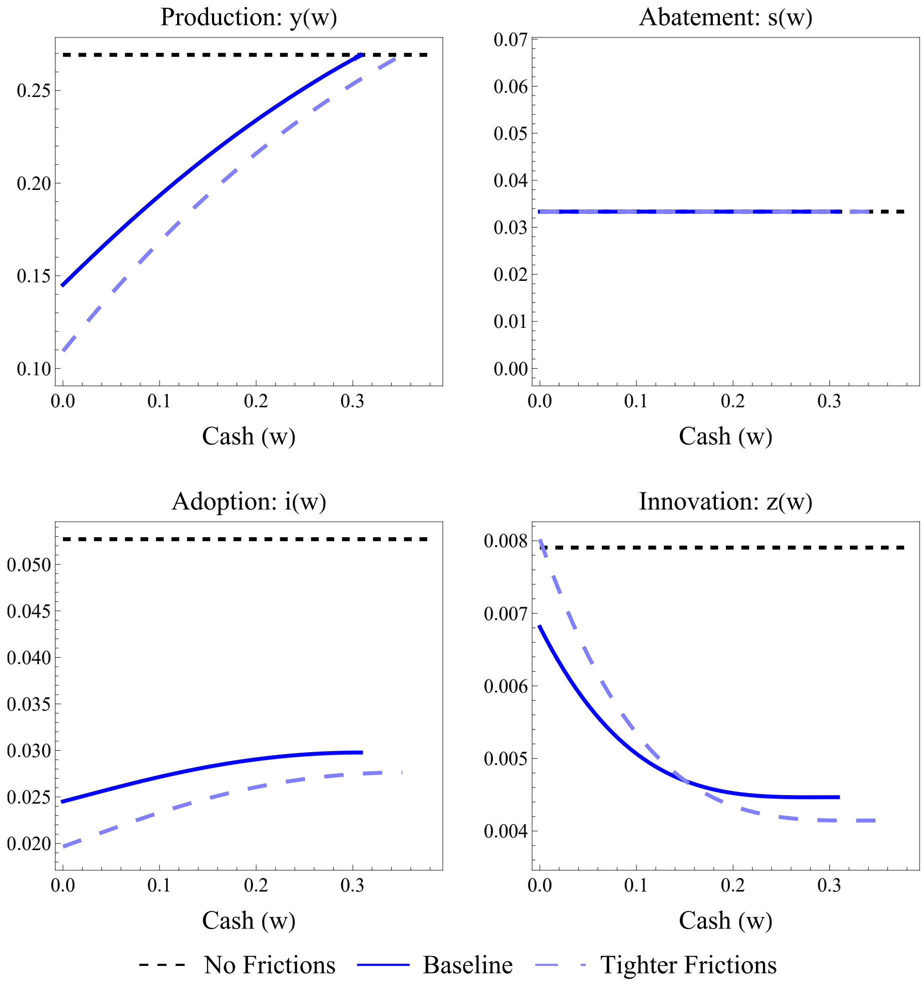

Figure 2: Optimal policies. The figure shows the optimal production y, abatement s, adoption i, and green innovation z as a function of cash reserves w. We consider the frictionless case with no financing frictions (black dashed line) against the financially constrained case when assuming relatively lower (solid blue line) and higher (light-blue dashed) financing costs.

Lemma 2. Moreover, the larger the cash reserves, the larger the firm's investment in adoption. As explained, financing frictions erode the marginal value of sustainability (then decreasing the benefit from adoption) and increase the value of cash (then increasing the effective cost of adoption), overall unambiguously reducing the optimal investment in adoption. I.e., all else equal, adoption is largely considered by the least financially constrained firms in the cross section, consistent with Accetturo et al. (2023) and De Hass et al. (2025).

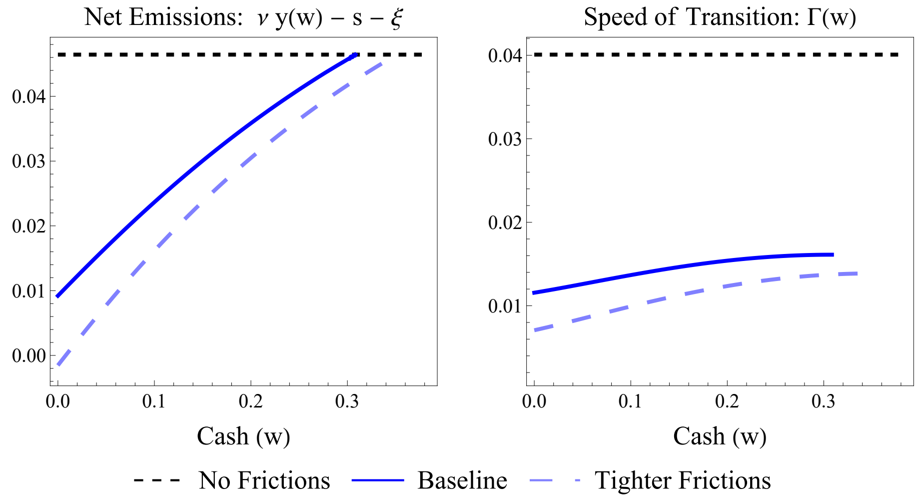

Figure 3: Carbon emissions and Sustainability. The figure shows the firm's net carbon emissions and the expected increase in sustainability as a function of cash reserves w. We consider the frictionless case with no financing frictions (black dashed line) against the financially constrained case when assuming relatively lower (solid blue line) and higher (light-blue dashed) financing costs.

Finally, the bottom right panel of Figure 2 shows the firm's optimal investment in green innovation. As discussed in Section 5, financing frictions increase *both* the effective benefit and cost of green innovation.[18] As a result, the impact on the optimal innovation rate is ambiguous. Under our baseline parameterization, the innovation rate decreases with cash reserves and always lies below z^*. However, the relative magnitude of the solid and dashed blue lines illustrate that financing frictions have a non-monotonic impact on the optimal innovation rate, which can become larger than in the benchmark case with no frictions. Below we investigate in more detail the sensitivity of green innovation to cash reserves (see Figure 4 and the associated discussion).

Mapping optimal policies to green outcomes Figure 3 shows the outcomes associated with these optimal choices by studying both current carbon emissions as well as the firm's green transition rate (equivalently, the speed at which the firm shifts to greener technologies). The left panel shows that the firm exhibits the lowest carbon emissions in the

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[18] Differently, recall that which financing frictions *decrease* the benefit of adoption and increase the cost.

case in which financing frictions are the highest, consistent with the result in Corollary 5. This is environmentally good. In turn, the right panel shows that financing frictions bear a negative impact on the speed of transition under our baseline parameterization, being the lowest in the case in which financing frictions are the largest. This is environmentally bad. In fact, the possibility of a pickup in the firm's innovation engagement in the presence of financing frictions is not enough to offset the decreased engagement in adoption.

Interestingly, our analysis reveals three notable points. First, different types of green investment exhibit a different exposure to financing frictions. Sorting green investment by its impact on the firm's sustainability G_t from the least to the most path-breaking, the model shows that financing frictions have a non-monotonic impact on green investment. In fact, the type of investment that is unambiguously negatively affected is adoption, i.e., the intermediate type. Second, financing frictions lead to an immediate reduction in the firm's current emissions, largely driven by a decline in the firm's production rate—and, thus, gross emissions. Third, the impact on financing frictions on the green transition rate is potentially ambiguous. In fact, it depends on whether the decrease in the firm's adoption rate is more than offset by the increase in the firm's engagement in green innovation. Under our benchmark parameterization—in which adoption is realistically cheaper than green innovation—we find that the negative impact dominates.

The drivers of firm's green investment under financing frictions. The analysis in the previous paragraph illustrates that financing frictions have an ambiguous effect on green innovation, which has direct implications for the speed of transition $\Gamma(w)$ (see equation 24). In this section, we further investigate the effect of cash flow volatility as analyzed in Proposition 4.

Confirming our analytical results, Figure 4 shows that green innovation exhibits a positive sensitivity to cash reserves if σ is sufficiently low. In that case, the firm holds significantly less cash, and green innovation increases with cash reserves. In turn, if σ is sufficiently high (in which case the firm increases its cash reserves substantially), the green

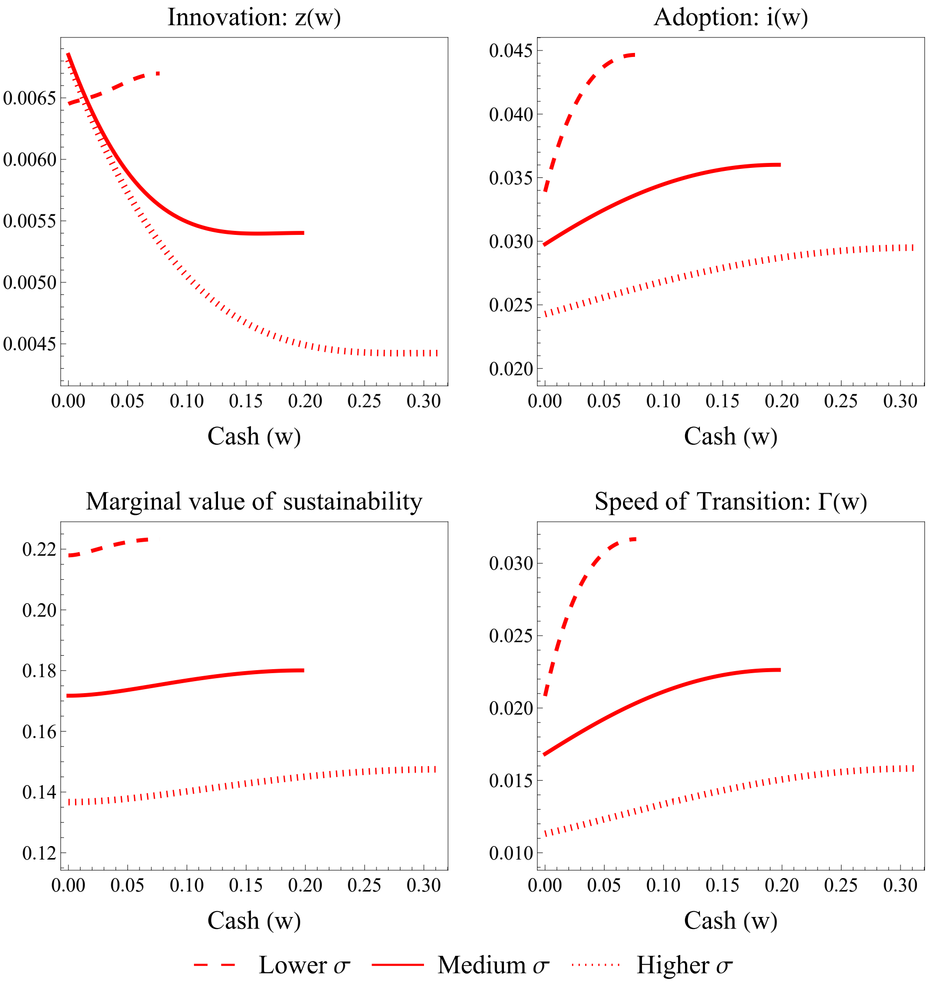

Figure 4: Green Investment and Cash flow Volatility The figure shows the firm's optimal green innovation, adoption, the marginal value of sustainability, and the transition rate as a function of cash reserves w and for different levels of cash flow volatility.

innovation rate becomes decreasing with cash reserves. As explained in the context of Proposition 4, the positive or negative sensitivity of innovation to cash is driven by how cash flow volatility differently affects the sensitivity of the benefit (the numerator of equation (17)) and of the effective cost of innovation (the denominator of equation (17)) to cash.

The top right panel shows that firms with more volatile cash flows invest less in adoption. Indeed, in the presence of financing frictions, the firm is effectively risk averse as firm value is concave in cash reserves. As a result, higher cash flow volatility reduces firm value. In

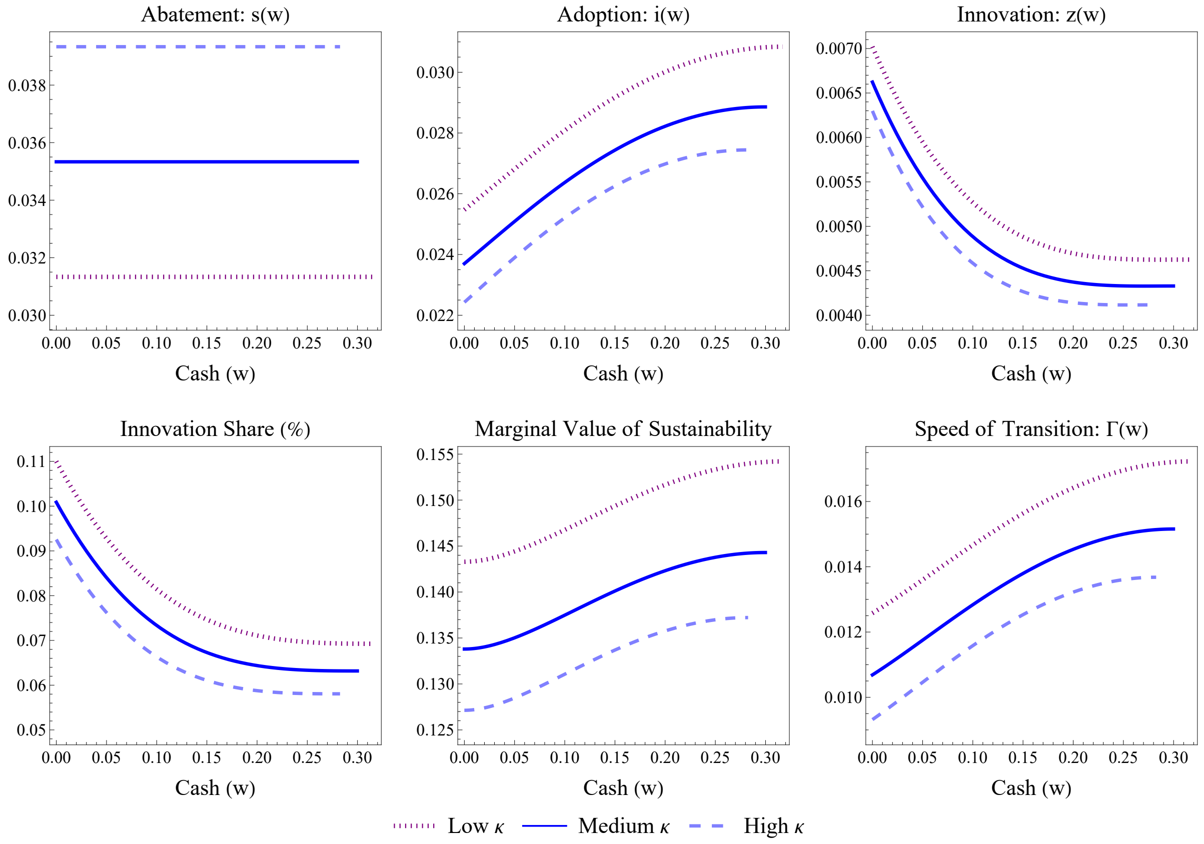

Figure 5: Carbon pricing and carbon emission management. The figure shows the firm's optimal abatement ($s(w)$), adoption ($i(w)$), the innovation share and the marginal value of sustainability as a function of the firm's cash reserves w and for different levels of carbon pricing κ.

turn, lower firm value depresses the marginal value of sustainability (as illustrated in the bottom left panel) and so the firm's engagement in adoption.

On net, the drop in the adoption rate exhibited by firms with volatile cash flows largely drives the green transition rate to be slower for these firms (see the bottom right panel). Thus, even if financing frictions can incentivize firms to engage more in green innovation thanks to the upside that green breakthroughs bring along, we find that it does not speed up the firm's transition rate for realistic parameterizations.

Carbon pricing, green investment, and financing frictions Figure 5 looks at the impact of carbon pricing κ on the firm's optimal green investment mix. A greater carbon tax increases the benefit from abatement while leaving the associated cost unchanged. In fact, abatement enables the firm to immediately reduce carbon emissions and, thus, curtails the firm's carbon tax liability. Thus, a higher carbon price κ leads the firm to increases its investment in abatement (top left chart).

At the same time, Figure 5 also shows that a greater κ leads to a reduction in the firm's engagement in adoption and in green innovation (top right and bottom left chart). Indeed, carbon pricing reduces the value of polluting firms (i.e., firms exhibiting positive net emissions) by imposing a larger tax liability on firms. This drop leads to a reduction in the marginal value of sustainability (see bottom middle panel). That is, perhaps surprisingly, carbon pricing decreases the firm's incentives to improve its degree of sustainability.

As illustrated in the bottom right chart, carbon pricing then leads to a reduction in the innovation share, defined as the weight of green innovation over the total engagement in green investment (defined as the sum of abatement, adoption, and green innovation). This result implies that firms shift to abatement and away from green innovation as carbon pricing becomes more costly. That is, firms tap green investments to reduce their emissions immediately rather than those that effective change the firms' technology and, thus, improve its sustainability. This result is consistent with Bustamante and Zucchi (2024a), who derive it as a setting in which firms are financially unconstrained.

While both imposing a financial burden on the firm, it is worth noting that financing frictions and carbon pricing have a distinct impact on the firm's optimal carbon emission management. Namely, an increase in carbon pricing leads to an increase in abatement and a drop in both adoption and green innovation—i.e., a higher κ leads to an unambiguous shift towards short-term measures (and away from green innovation), as predicted by Bustamante and Zucchi (2024a). In turn, an increase in financing frictions has no impact on abatement, a drop in adoption, and an ambiguous (potentially positive) impact on green innovation.

6 The impact of socially responsible capital

A key virtue of our model is that it studies the impact of financing frictions on green outcomes by capturing the real-world richness of policies that firms can undertake to reduce their carbon emissions and improve their sustainability. Thus, our model provides a suitable framework to investigate how firms respond to socially responsible investment—i.e., investment aimed at combating climate change. In this context, two prevalent strategies are exclusion and tilting, which we analyze next.

6.1 Exclusion

We start by investigating the impact of a common strategy adopted by socially responsible investors: Exclusion, which involves starving polluting firms of funding. If firms are indeed excluded by investors, they need to rely on their internally-generated financial resources that they store in the cash reserves. Whenever they deplete their cash reserves, they are forced into liquidation. Through the lens of our model, we investigate the impact of exclusion at the firm level by studying the difference in optimal policies when the firm is perspectively forced into liquidation at $w = 0$ as opposed to being able to refinance.[19]

Figure 6 plots such differences when looking at selected optimal choices and outcomes. Because internally-generated resources are the main source of funding for the firm under an exclusion strategy, such a strategy gives rise to a sharp increase in the marginal value of cash, which erodes the marginal value of sustainability. In turn, the reduction in the marginal value of sustainability disincentivizes the firm to invest in adoption, which sharply declines too.[20]

At the same time, the exclusion strategy has an ambiguous impact on green innovation—consistent with the result in our baseline analysis that financing frictions have an ambiguous impact on this type of green investment. Because a green breakthrough signals the firm's

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[19] Recall that if the firm liquidates at zero, then the relevant boundary condition is equation (23).

[20] Because the optimal abatement rate is not affected by the severity of financial constraints, it is not affected by exclusion strategies (then, it is not shown in the figure).

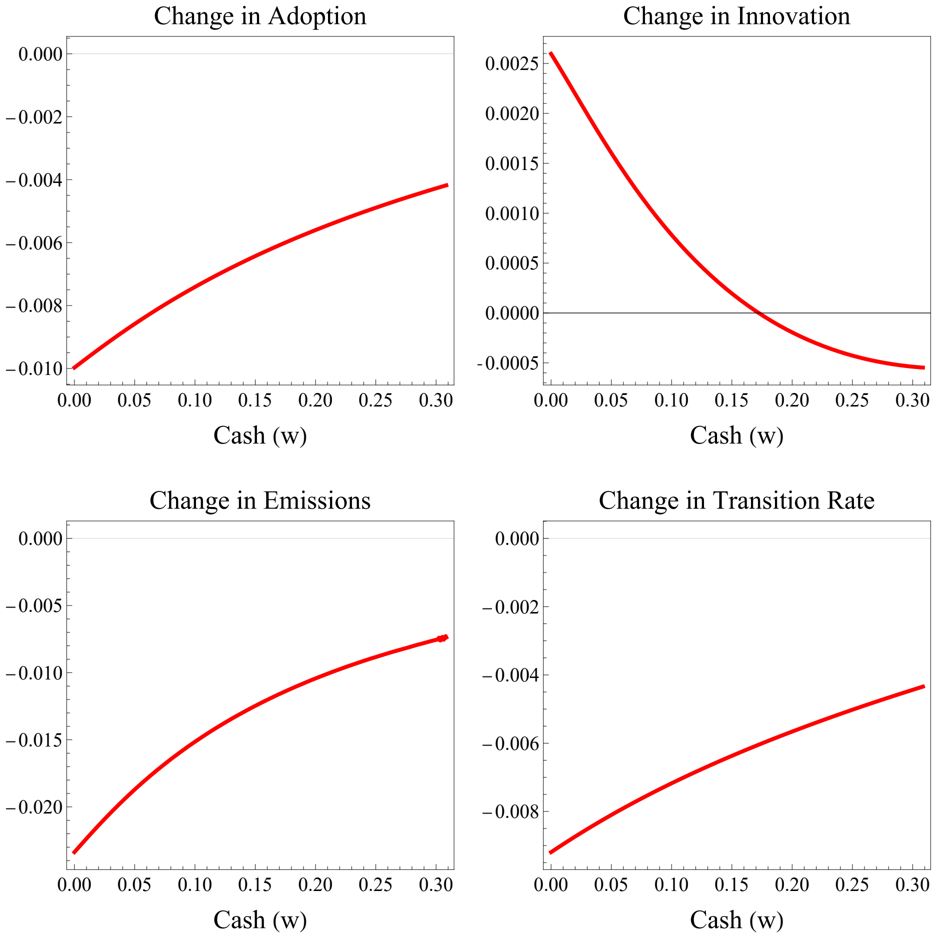

Figure 6: THE IMPACT OF EXCLUSION STRATEGIES ON GREEN INVESTMENT AND THE ENVIRONMENT. The red solid line captures the effect of an exclusion strategy on the firm's engagement in adoption and green innovation (top panels), as well as the associated environmental impact in terms of carbon emissions and the transition rate (bottom panels). All quantities are expressed as absolute changes as a function of cash reserves.

commitment to becoming more sustainable, it provides the firm with an opportunity to access fresh financing. As a result, the benefit associated with green innovation is greater under an exclusion strategy. Yet, because the associated cost is also greater under an exclusion strategy (as so does the marginal value of cash), the impact of exclusion strategy on green innovation is ambiguous. Under our baseline parameterization, Figure 6 shows that the firm's engagement in green innovation might increase if cash reserves are sufficiently low, and decreases otherwise.

The bottom panels of Figure 6 show the environmental outcomes associated with exclusion strategies, captured by current emissions and the speed of transition. Exclusion strategies block access to external funds and, thus, increase the severity of financial constraints and the firm's effective risk aversion. Thus, the firm then finds it optimal to reduce its optimal scale of production to reduce its cash flow volatility. The lower scale of production, in turn, translates into lower carbon emissions, as illustrated in the bottom left panel. At the same time, however, exclusion strategies slow down the transition to greener technologies due to the sharp negative impact on green adoption. Indeed, the reduced engagement in adoption more than offsets the greater engagement in green innovation. This result resonates with the findings in the theoretical study by Edmans, Levit, and Schneemeier (2023), who also question the effectiveness of exclusion strategies.

Overall, our analysis suggests that exclusion strategies can indeed lower emissions in the short term thanks to a lower production rate. This is environmentally good. However, and worringly, these strategies slow down the speed of green transition by reducing firms' marginal value of sustainability. This is environmentally bad. If limiting climate change can only be attained by developing greener technologies—as emphasized by the United Nations' Intergovernmental Panel on Climate Change (see IPCC, 2022)—our analysis then suggests that exclusion strategies may backfire.

6.2 Tilting

Tilting is a strategy in which investors ease the terms of financing to a polluting firm as long as it exhibits some engagement to becoming greener. Through the lens of our model, this means that financing becomes cheaper for firms that exhibit a commitment to increase their degree of sustainability, G_t.

To capture tilting in a parsimonious fashion, we then assume that the fixed cost of financing is constant as opposed to scaling with G_t. This means that equity issuance costs gradually decrease as the firm's degree of sustainability improves. As the firm's sustainability increases, the firm would effectively outgrow the fixed issuance cost and its

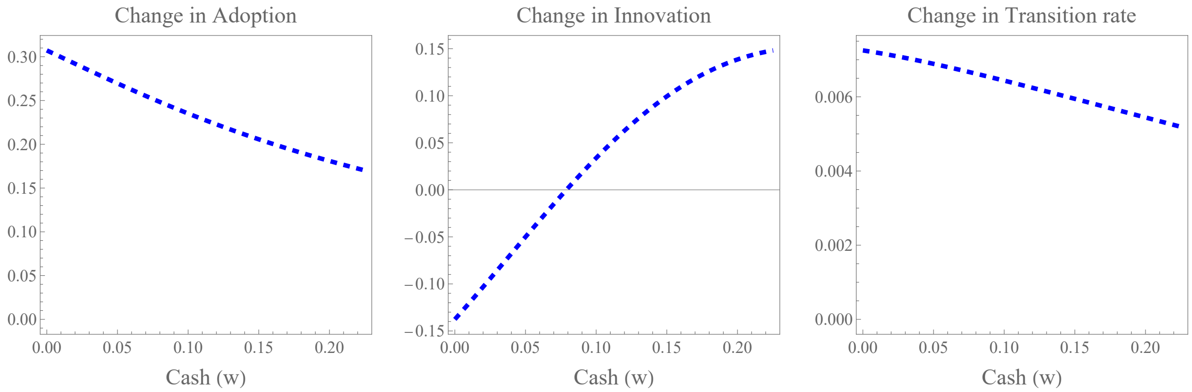

Figure 7: The impact of tilting on innovation and the transition rate. The dashed blue line captures the effect of a tilting strategy on the firm's adoption of green technologies (left panel), engagement in innovation (middle panel) and its transition rate to sustainable technologies (right panel). All quantities are expressed as absolute changes as a function of cash reserves.

optimal policy would converge to the one of a firm facing proportional issuance costs only. In this case, the dynamics of firm value continue to satisfy equation (33) for any $w < W^*$. Thus, the optimal policies derived in Section 4 continue to have the same functional form, and so the boundary conditions at w^* (i.e., see equation (18) and (19)). Yet, the boundary conditions at zero is different and given by:

$$v'(0) = 1 + \chi, \tag{25}$$

which means that when cash reserves are depleted, the marginal value of cash (the left-hand side) equals the marginal cost of funding (the right-hand side). That is, the firm refinances every time the cash reserves are depleted by issuing an infinitesimal amount that reflects the cash process back into the cash retention region.

Consider first the impact of tilting on the firm's incentives to invest in adoption. Because tilting implies a lower financing cost as long as the firm's sustainability increases, it makes sustainability more valuable. As a direct implication of the increased marginal value of sustainability, tilting thus leads the firm to invest more in the adoption of greener

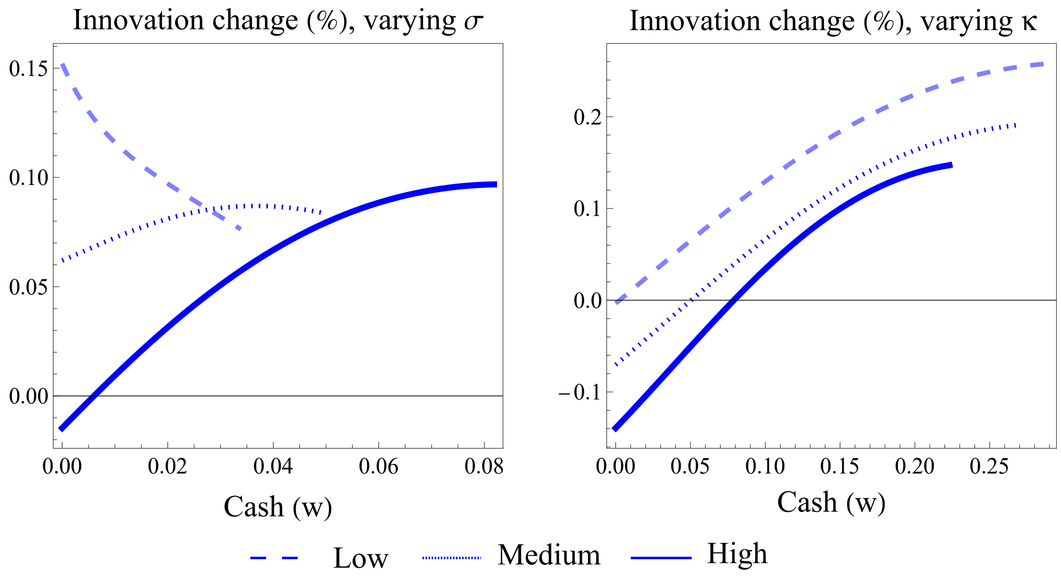

Figure 8: Tilting and Innovation: Comparative Statics. The figure shows the impact of tilting strategies on green innovation for firms with various degrees of either cash flow volatility (σ left panel) or carbon prices (κ, right panel). All quantities are expressed in percentage changes, and as a function of cash reserves.

technologies.[21]

Consider now the impact of tilting on innovation. Consistent with the prediction that financing frictions have an ambiguous impact on innovation, the impact of tilting on green innovation is ambiguous, as illustrated in the middle panel of Figure 7. Figure 8 further zooms on innovation and shows that the effect of tilting is positive for firms with low cash flow volatility. Recall that low-volatility firms are those reducing their engagement in green innovation in the presence of financing frictions, and more so when holding low cash reserves. Thus, these firms invest more in green innovation when financing frictions loosen thanks to tilting strategies. Moreover, as a higher carbon tax discourages innovation (as discussed in previous sections and consistent with Bustamante and Zucchi (2024a)), the right panel of Figure 8 shows that tilting has a positive effect on green innovation if firms operate in a jurisdiction where carbon prices are sufficiently low.

Driven by the positive impact on adoption and despite the potentially-ambiguous effect

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[21] As for exclusion, the optimal abatement rate is not affected by tilting (not shown in the figure).

on innovation, tilting strategies lead to an increase in the speed of transition under our baseline parameterization, as shown in the right panel of Figure 7. More generally, this is the case when the transition is mainly adoption-based as opposed to innovation-based, which is the case when adoption is sufficiently cheaper than green innovation (as is the case in reality). Overall, our analysis then illustrates that tilting accelerates the firm's transition to more sustainable production processes, although it may do so mostly through adoption as opposed to green innovation. The ambiguous and potentially negative impact of tilting on green innovation is consistent with the findings of Atta-Darkua, Glossner, Krueger, and Matos (2022), who fail to find evidence that climate conscious investors seek companies active in the development of green patents.[22]

7 The impact of subsidies

To foster the green transition, regulators around the world have discussed (or implemented) subsidies to mitigate financial barriers that could discourage green investment. Subsidies can be broadly classified into two categories. The first represents subsidies to incentivize firms to adopt existing greener technologies (like switching the truck fleet to electric). The second represents subsidies that support firms' investment in green innovation—i.e., into major breakthroughs that can effectively lead to a decarbonized economy. In this section, we investigate the impact of such subsidies through the lens of our model.

Subsidies to adoption Subsidies to the adoption of green technologies have been endorsed by several countries around the world.[23] For a given adoption rate, subsidies reduce the associated cost to $\frac{\theta(1-\psi)i_t^2}{2}G_t$, where ψ represents the magnitude of the subsidy. As a

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[22] Relatedly, Noh, Oh, and Song (2023) show that equity investors do not seem to care about the production of green patents.

[23] For instance, Accetturo et al. (2023) report the case of Italy.

result, the optimal investment in adoption satisfies:

$$i(w) = \frac{1}{\theta(1-\psi)v'(w,\psi)}\left[v(w,\psi) - wv'(w,\psi)\right] \tag{26}$$

The direct impact of the subsidy is to reduce the cost associated with adoption. At the same time, however, the subsidy also impacts firm value and the marginal value of cash. Thus, it should also affect the marginal value of sustainability and the firm's incentives to invest in innovation.

Figure 9 shows indeed the impact of such subsidy. Not surprisingly, it leads firms to increase their investment in adoption. At the same time, it also leads to an increase in the firm's engagement in green innovation. Indeed, these subsidies increase firm value and, thus, the marginal value of sustainability too (see Lemma 1). Thus, these subsidies also increase the benefit from green innovation, which also increase in the presence of such subsidies. Whereas current carbon emissions do not decrease much (not shown in the figure), the expected increase in sustainability ($\Gamma(w)$) unambiguously increases in the presence of such subsidies. As adoption increases by more, however, the innovation share decreases following the introduction of the subsidy.

Overall, our results suggest that subsidies can "kill two birds with one stone": By boosting the marginal value of sustainability, they not only effectively increase adoption (i.e., the investment that they effectively target) but they also increase the firm's engagement in green innovation.

Subsidies to green innovation We now consider another type of subsidy, namely, those targeting green innovation. Similar to those targeting adoption, they work as a rebate in the effective cost of innovation. That is, an ex-ante subsidy abates the cost of innovation to $\frac{\zeta}{2}(1-\iota)z^2G_t$, where ι represents the magnitude of the subsidy. Under these assumptions, the optimal innovation policy boils down to:

$$z(c) = \frac{\phi}{\zeta(1-\iota)v'(w,,\iota)}\left[\lambda v(w^*,\iota) - v(w,\iota) - (\lambda w^*(\iota) - w)\right]. \tag{27}$$

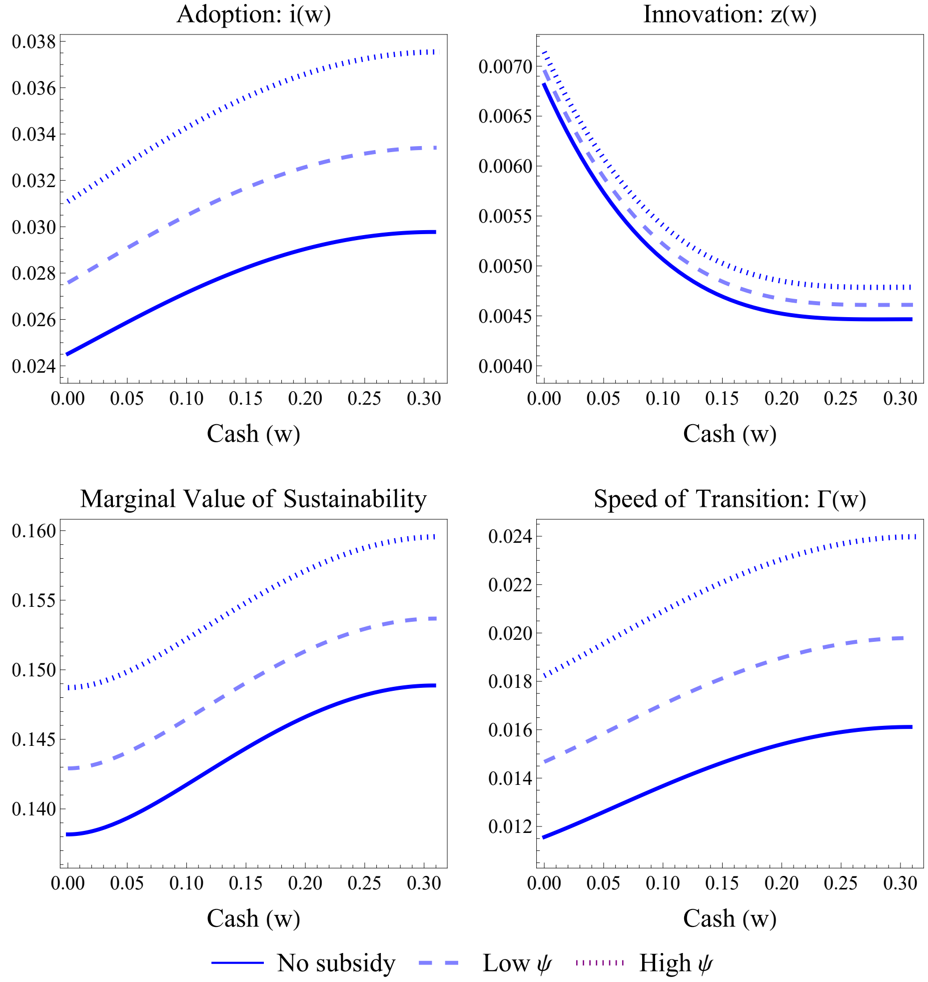

Figure 9: The impact of subsidies to adoption. The figure shows the impact of subsidies to adoption on the firm's investment in adoption, green innovation, on the marginal value of sustainability, and on the speed of transition $\Gamma(w)$. The solid line represents the case with no subsidy, the dashed line represents the case with low subsidy (8%), whereas the dotted line represents the cash with high subsidy (15%).

By effectively decreasing the cost of innovation, the firm's optimal innovation rate should increase. However, as the subsidy also affects firm value, the firm's incentives to invest in adoption, and the optimal cash hoarding choices, the impact on the firm's innovation share is not a priori obvious.

Figure 10 shows the impact of green subsidies. On top of the case with no subsidies (solid line), it considers subsidies to green innovation ι of the same magnitude as those

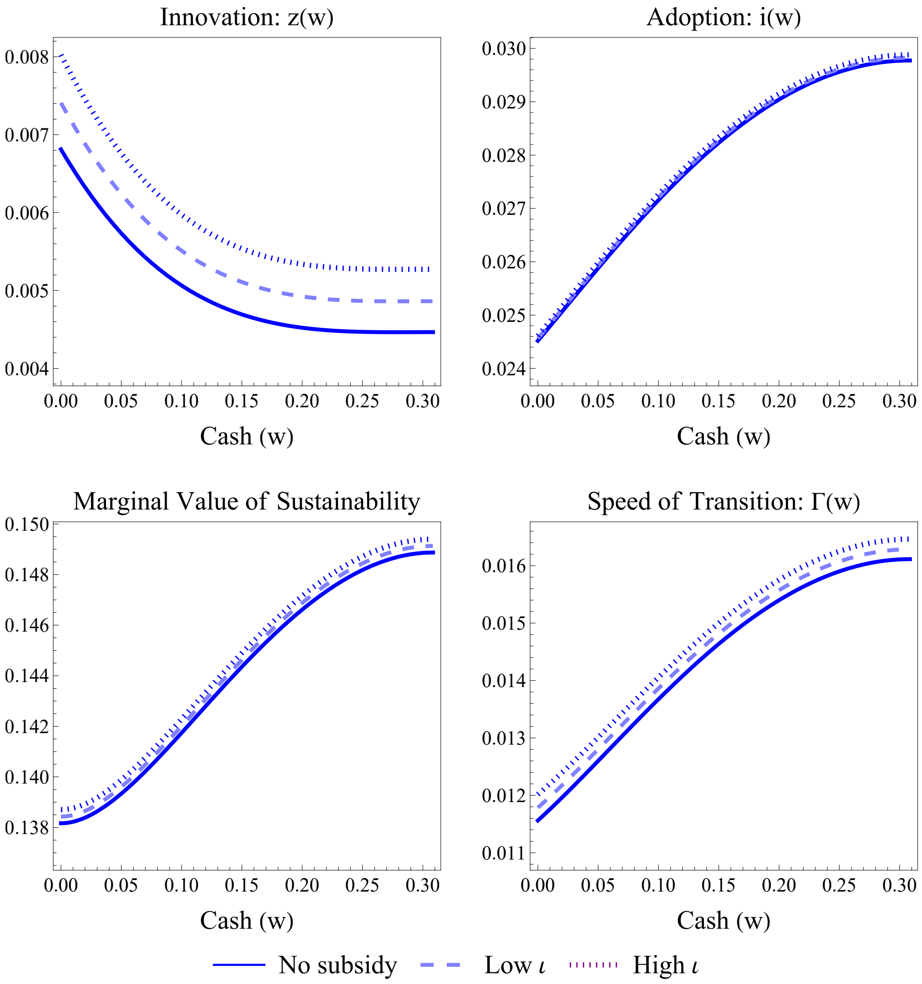

Figure 10: THE IMPACT OF SUBSIDIES TO GREEN INNOVATION. The figure shows the impact of subsidies to green innovation on the firm's investment in green innovation, adoption, the marginal value of sustainability, and the speed of transition $\Gamma(w)$. The solid line represents the case with no subsidy, the dashed line represents the case with low subsidy (8%), whereas the dotted line represents the cash with high subsidy (15%).

considered in Figure 9 (in that figure, however, we explore the impact of subsidies to adoption). Obviously, this type of subsidy leads the firm to increase its engagement in green innovation. The top right panel also shows that subsidies to innovation stimulate adoption albeit only by a little, due to a slight increase in the marginal value of sustainability.[24]

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[24] Note that the relative impact of subsidies to adoption and innovation on the opposite quantity relies on the idea that adoption is cheaper than innovation.

8 Concluding remarks

We develop a novel theoretical framework to investigate how financing frictions affect the firm's carbon emissions as well as the transition rate to more sustainable technologies. As a novelty compared to previous contributions, we allow for a realistically rich description of green investment types, which allows us to capture the dual dimension of combating climate change: The reduction in carbon emissions in the short run and the transition to greener technologies in the medium to long-term. Our model predicts that financing frictions lead to a contraction in the firm's scale of production, and impact each type of green investment differently—namely, when sorting these investments by their effect on technological sustainability, the impact is non-monotonic. Our analysis shows that financing frictions have no impact on the abatement of current emissions, a negative impact on the adoption of existing technologies, and an ambiguous impact on green innovation.

As an important take-away, our model then implies that financing frictions reduce current emissions—which is good for the environment—but have a negative impact on the speed of green transition as long as this is mainly adoption-based. Our model also provides a tool to understand the impact of initiatives aimed at supporting green investment, such as socially-responsible investment and public subsidies. Our analysis shows that that exclusion strategies curb emissions but slow down the transition to greener technologies, whereas tilting strategies speed up the green transition if it is adoption-based although have ambiguous impact on green innovation. Moreover, subsidies aimed at supporting the adoption of greener technology may boost green innovation too, potentially killing two birds with one stone.

A Appendix

A.1 Proof of the results in section 3

In this Appendix, we prove the result regarding the benchmark with no financing frictions (i.e., $\chi = 0$ and $f = 0$). Using the conjecture that firm value scaled with sustainability gives the following scaled HJB:

$$\rho\, v^* = \max_{y^*, s^*, i^*, z^*} \; p(y^*)y^* - \frac{(s^*)^2\alpha}{2} - \frac{(z^*)^2\zeta}{2} - \frac{(i^*)^2\theta}{2} - \kappa\left(y^*\nu - s^* - \xi\right) + (i^* - \delta)v^* + \phi z^* \left[\lambda - 1\right] v^* \tag{28}$$

Substituting the optimal policies into the above equation gives

$$\frac{1}{2}\left(\frac{1}{\theta} + \frac{\phi^2(\lambda-1)^2}{\zeta}\right)(v^*)^2 - (\delta+\rho)v^* + \frac{(a-\nu\kappa)^2}{4b} + \frac{\kappa^2}{2\alpha} + \xi\kappa = 0 \tag{29}$$

so we have that firm value absent frictions solves equation (11). Differentiating with respect to κ gives

$$\frac{dv^*}{d\kappa} = \frac{\xi + \frac{\kappa}{\alpha} - \frac{\nu(a-\nu\kappa)}{2b}}{\sqrt{(\delta+\rho)^2 - 2\left(\frac{1}{\theta} + \frac{\phi^2(\lambda-1)^2}{\zeta}\right)\left(\frac{(a-\nu\kappa)^2}{4b} + \frac{\kappa^2}{2\alpha} + \xi\kappa\right)}}$$

which is negative as long as the firm exhibits negative emissions (i.e., note that the numerator is simply negative emissions).

A.2 Proof of the results in section 4

Consider now the model with financing constraints in Section 4. Using our conjecture that firm value scales with sustainability gives:

$$
\begin{aligned}
V_G &= v - Gv'(w)\frac{C}{G^2} = v - wv' && (30)\\
V_W &= v' && (31)\\
V_{WW} &= \frac{v''}{G}. && (32)
\end{aligned}
$$

Thus, the scaled HJB equation satisfies:

$$
\begin{aligned}
\rho v(w) = \max_{y,s,i,z} \; & [rw + p(y)y - \frac{s^2\alpha}{2} - \frac{z^2\zeta}{2} - \frac{i^2\theta}{2} - \kappa(\nu y - s - \xi)]v' + \frac{\sigma^2}{2}y^2 v'' \\
& + (i-\delta)(v - wv') + \phi z\left[\lambda v(w^*) - v(w) - (\lambda w^* - c)\right] \qquad (33)
\end{aligned}
$$

Differentiating the above equation with respect to y, s, i, and z gives the optimal policies reported in the main text. Substituting the optimal policies into the above HJB gives the following equation

$$
\begin{aligned}
(\rho+\delta)v(w) =& \frac{\phi^2}{2\zeta v'}\left[\lambda v(w^*) - v(w) + w - \lambda w^*\right]^2 + \frac{[v(w) - wv'(w)]^2}{2\theta v'} + \frac{[a-\nu\kappa]^2 (v')^2}{2(2bv' - \sigma^2 v'')} \\
& + \frac{\kappa^2}{2\alpha}v' + (r+\delta)wv' + \xi\kappa v'
\end{aligned}
$$

which can be rewritten as

$$
\begin{aligned}
(\rho+\delta)\left[v - wv'\right] =& \frac{\phi^2}{2\zeta v'}\left[\lambda v(w^*) - v(w) + w - \lambda w^*\right]^2 + \frac{[v(w) - wv'(w)]^2}{2\theta v'} + \frac{[a-\nu\kappa]^2 (v')^2}{2(2bv' - \sigma^2 v'')} \\
& - (\rho - r)wv' + \frac{\kappa^2}{2\alpha}v' + \xi\kappa v'
\end{aligned}
$$

Now, we define $\Omega \equiv v(w^*) - w^*$—i.e., the enterprise value at the target cash level w^* in the presence of financing frictions. By calculations, we get:

$$\frac{1}{2}\left(\frac{1}{\theta} + \frac{\phi^2\left(\lambda-1\right)^2}{\zeta}\right)\Omega^2 - (\delta+\rho)\Omega + \frac{\left(a-\nu\kappa\right)^2}{4b} + \frac{\kappa^2}{2\alpha} + \xi\kappa - (\rho-r)w^*.$$

so

$$\Omega = \frac{(\delta+\rho) - \sqrt{(\delta+\rho)^2 - 2\left(\frac{1}{\theta}+\frac{\phi^2(\lambda-1)^2}{\zeta}\right)\left(\frac{(a-\nu\kappa)^2}{4b}+\frac{\kappa^2}{2\alpha}+\xi\kappa-(\rho-r)w^*\right)}}{\left(\frac{1}{\theta}+\frac{\phi^2(\lambda-1)^2}{\zeta}\right)}$$

and then we have

$$v(w^*) = w^* + \frac{(\delta+\rho) - \sqrt{(\delta+\rho)^2 - 2\left(\frac{1}{\theta}+\frac{\phi^2(\lambda-1)^2}{\zeta}\right)\left(\frac{(a-\nu\kappa)^2}{4b}+\frac{\kappa^2}{2\alpha}+\xi\kappa-(\rho-r)w^*\right)}}{\left(\frac{1}{\theta}+\frac{\phi^2(\lambda-1)^2}{\zeta}\right)} \tag{34}$$

Note that this expression is very similar to equation (11) absent financing frictions but for the presence of the term w^*—i.e., the optimal amount of cash held by the firm.

Proof of Lemma 1 As just derived in the above calculations, the marginal value of sustainability is $V_G = v - wv'$ in the case with financing frictions, whereas it is $V_G^* = v^*$ in the benchmark case with no frictions case. Thus, if financing frictions erode firm value, i.e. $v(w) < v^*$, then indeed $V_G < V_G^*$ as the marginal value of cash v' should always be greater than one—a standard result in cash management models. ■

Proof of Lemma 2 Consider claim (1) about the optimal production rate. Note that the production rates with and without financing frictions coincide at $w = w^*$—i.e., $y(w^*) = y^*$—due to the boundary conditions at the target cash level. Dividing the numerator and the denominator of $y(w)$ by v' then illustrates that $y^* \geq y(w)$ for any w as long as financing frictions make the firm effectively risk averse as $-v''/v' > 0$—a standard result in dynamic cash management models.

Claim (2) about the optimal abatement rate is straightforward given that the optimal abatement rate exhibits the same expression irrespective of the presence of financing frictions.

Claim (3) about adoption is straightforward given claim (1) and $v' \geq 1$—a standard result in cash management models. That is, in the presence of financing frictions, the numerator of $i(w)$ decreases whereas its denominator increases—leading to an unambiguous decrease in adoption.

Lastly, consider claim (4). At $w = w^*$, the optimal innovation rate $z(w^*)$ is lower than z^* if $v^* > v(w^*) - w^* \equiv \Omega$ (as defined above). Using equation (11) and equation (34), this inequality boils down to $(\rho - r)w^* > 0$. By assumption, $\rho - r > 0$ to make sure that there is a cost associated with holding cash (so that the firm has no incentive to accumulate an infinite cash reserves). Moreover, if the firm faces financing costs, $w^* > 0$. Thus, $z(w^*) < z^*$. Consider now the optimal innovation rate at lower levels of cash reserves. Consider the denominator of the optimal innovation rate, representing the effective cost of green innovation. In the presence of financing frictions, this is given by $\zeta v'(w)$ which, as long as cash is more valuable inside the firm than if paid out ($v' > 1$, which is standard in cash management models), is greater than the denominator absent financing frictions (which is simply equal to ζ). The numerator of $z(w)$ (representing the effective benefit from green innovation) is given by $\phi\left[\lambda v(w^*) - v(w) - (\lambda w^* - w)\right]$ at a given cash level in the presence of financing frictions (whereas it is equal to $\phi\left[\lambda - 1\right] v^*$ absent financing frictions). Notably, the numerator of $z(w)$ is decreasing with cash reserves—i.e., the first derivative of the numerator is $\phi(1 - v')$, which is negative whenever cash is more valuable inside the firm than if paid out, $v' > 1$. Thus, the benefit from green innovation is bigger for lower level of cash reserves, as a green breakthrough allows the firm to access fresh financing at no cost. At $w = 0$, the benefit from green innovation boils down to $\phi\left[\lambda\Omega - \ell\right]$ if the firm liquidates.[25] If ℓ is sufficiently smaller than v^*, then the benefit from green innovation is greater than in the benchmark case with no frictions.[26] This greater benefit can more than

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[25] Note that we focus on the liquidation case as it represents the case in which financing frictions are the tightest.

[26] As discussed above, $\Omega < v^*$ as long as the opportunity cost of cash is strictly positive.

offset the greater cost, in which case the optimal innovation rate is greater in the presence of financing frictions. (We also investigate this pattern numerically, see section 5.2. ■

Proof of Lemma 3 To prove claim (1), we simply differentiate $\frac{d(v-wv')}{dw} = -wv''$, which is positive if the firm is effectively risk averse.

Claim (2) follows from the fact that the abatement policy is independent of w.

Claim (3) stems by differentiating $i(w)$ with respect to w, which gives

$$\frac{d}{dw}i(w) = -\frac{v(m)v''(m)}{\theta(v')^2} \tag{35}$$

Claim (4) about green innovation builds on the result that both the effective benefit (the numerator) of $z(c)$ as well as its effective cost (the denominator) decrease with the level of cash reserves. In fact, the first derivative of the numerator is negative as long as cash is more valuable inside the firm than outside (i.e., $v' \geq 1$), and the first derivative of the denominator is negative as the firm is effectively risk averse. (Proposition 4 clarifies when one pattern or the other can dominate.) ■

Proof of Proposition 4 The ODE describing the dynamics of firm value can be rewritten as

$$(\rho+\delta)v = \frac{\zeta}{2}z^2v' + \frac{\theta}{2}i^2v' + \frac{\alpha}{2}s^2v' + \frac{(a-\nu\kappa)}{2}yv' + wv'(r+\delta) + \xi\kappa v'$$

Differentiating the expression with respect to w, and rewriting it:

$$2\zeta zz' = 2\lambda - 2\theta ii' - (a-\nu\kappa)y' + \frac{v''}{v'}\left[-2w(r+\delta) - \frac{\kappa^2}{\alpha} - 2\kappa\xi - \theta i^2 - (a-\nu\kappa)y - \zeta z^2\right]$$

which by replacing $i'(w)$ and $y'(w)$ can be re-written as

$$\begin{aligned} 2\zeta zz' =& 2\lambda + 2\frac{vv''(v-wv')}{\theta(v')^3} - \frac{\sigma^2(a-\nu\kappa)^2(-(v'')^2+v'v''')}{(2bv'-\sigma^2v'')^2} && (36)\\ &+\frac{v''}{v'}\left[-2w(r+\delta) - \frac{\kappa^2}{\alpha} - 2\kappa\xi - \theta i^2 - (a-\nu\kappa)y - \zeta z^2\right] && (37)\end{aligned}$$

So, as $\lambda > 0$, $z' > 0$ if $\sigma \to 0$, in which case $v'' \to 0$. This proves the first part of the claim.

Given the claim 4 in Lemma 2, then if z is increasing, then z always lies below z^*. In turn, if z is decreasing, then we can only verify that the inequality holds in a left neighborhood of w^*. $\blacksquare$

References

Accetturo, A., G. Barboni, M. Cascarano, E. Garcia-Appendini, and M. Tomasi. 2023. "Credit Supply and Green Investments."

Acemoglu, D., P. Aghion, L. Bursztyn, and D. Hemous. 2012. "The Environment and Directed Technical Change." *American Economic Review* 102:131–166.

Acemoglu, D., U. Akcigit, D. Hanley, and W. Kerr. 2016. "Transition to Clean Technology." *Journal of Political Economy* 124:52–104.

Acemoglu, D., and D. Cao. 2015. "Innovation by Incumbents and Entrants." *Journal of Economic Theory* 157:255–294.

Akcigit, U., and W. Kerr. 2018. "Growth Through Heterogenous Innovation." *Journal of Political Economy* 126:1374–1443.

Atta-Darkua, V., S. Glossner, P. Krueger, and P. Matos. 2022. "Decarbonizing Institutional Investor Portfolios."

Bartram, S., K. Hou, and S. Kim. 2022. "Real Effects of Climate Policy: Financial Constraints and Spillovers." *Journal of Financial Economics* 143:668–696.

Bolton, P., H. Chen, and N. Wang. 2013. "Market Timing, Investment, and Risk Management." *Journal of Financial Economics* 109:40–62.

—. 2011. "A Unified Theory of Tobin's q, Corporate Investment, and Financing." *Journal of Finance* 66:1545–1578.

Bowen, A., and N. Stern. 2010. "Environmental Policy and the Economic Downturn." *Oxford Review of Economic Policy* 26:137–163.

Bruegem, M., R. Marfe, and F. Zucchi. 2024. "Corporate Policies and the Term Structure of Risk."

Bugshan, A. 2022. "Oil Price Volatility and Corporate Cash Holdings." *Journal of Commodity Markets* 28:100237.

Bustamante, M., and F. Zucchi. 2024a. "Dynamic Carbon Emission Management." Working Paper.

—. 2024b. "Innovation, Industry Equilibrium, and Discount Rates." Working Paper.

Chiu, W., P.H. Hsu, K. Li, and T. Tong. 2024. "Green Products." Working Paper.

Choi, D., Z. Gao, W. Jiang, and H. Zhang. 2024. "Carbon Firm Devaluation and Green Actions." Working Paper.

Clementi, G., and H. Hopenhayn. 2006. “A Theory of Financing Constraints and Firm Dynamics.” *Quarterly Journal of Economics* 121:229–265.

Cohen, L., U. Gurun, and Q. Nguyen. 2025. “The ESG-Innovation Disconnect: Evidence from Green Patenting.” *Management Science*, Forthcoming.

De Hass, R., R. Martin, M. Muuls, and H. Schweiger. 2025. “Managerial and Financial Barriers during the Green Transition.” *Management Science*, Forthcoming.

Décamps, J.P., S. Gryglewicz, E. Morellec, and S. Villeneuve. 2017. “Corporate Policies with Permanent and Transitory Shocks.” *Review of Financial Studies* 30:162–210.

Décamps, J.P., T. Mariotti, J.C. Rochet, and S. Villeneuve. 2011. “Free cash flows, issuance costs, and stock prices.” *Journal of Finance* 66:1501–1544.

Della Seta, M., E. Morellec, and F. Zucchi. 2020. “Short-term Debt and Incentives for Risk-taking.” *Journal of Financial Economics* 137:179–203.

Derrien, F., P. Krueger, A. Landier, and T. Yao. 2023. “ESG News, Future Cash Flows, and Firm Value.” Working Paper.

Doettling, R., and M. Rola-Janicka. 2023. “Too Levered for Pigou: Carbon Pricing, Financial Constraints, and Leverage Regulation.” Working Paper.

Dumas, B. 1991. “Super-Contact and Related Optimality Conditions.” *Journal of Economic Dynamics and Control* 15:675–685.

Edmans, A., D. Levit, and J. Schneemeier. 2023. “Socially Responsible Divestment.” Working Paper.

Farre-Mensa, J., D. Hegde, and A. Ljungqvist. 2020. “What is a Patent Worth? Evidence from the U.S. Patent Lottery.” *Journal of Finance* 75:639–682.

Farre-Mensa, J., and A. Ljungqvist. 2016. “Do Measures of Financial Constraints Measure Financial Constraints?” *Review of Financial Studies* 29:271–308.

Glover, B. 2016. “The Expected Cost of Default.” *Journal of Financial Economics* 119:284–299.

Graham, J. 2000. “How Big are the Tax Benefits of Debt.” *Journal of Finance* 55:1901–1941.

Graham, J.R., and M.T. Leary. 2018. “The Evolution of Corporate Cash.” *Review of Financial Studies* 65:451–4469.

Graham, J.R., M.T. Leary, and M. Roberts. 2015. “A Century of Capital Structure: The Leveraging of Corporate America.” *Journal of Financial Economics* 118:658–683.

Hall, B.H. 2019. "Is there a role for patents in the financing of innovative firms?" *Industrial and Corporate Change* 28:657–680.

Hall, R. 2018. "New Evidence on the Markup of Prices over Marginal Costs and the Role of Mega-Firm in the US Economy." NBER Working Paper.

Hartzmark, S., and K. Shue. 2023. "Counterproductive Sustainable Investing: The Impact Elasticity of Brown and Green Firms." Working Paper.

Hege, U., S. Pouget, and Y. Zhang. 2023. "The Impact of Corporate Climate Action on Financial Markets: Evidence from Climate-Related Patents."

Heider, F., and R. Inderst. 2022. "A Corporate Finance Perspective on Environmental Policy." Working Paper.

Heinkel, R., A. Kraus, and J. Zechner. 2001. "The Effect of Green Investment on Corporate Behavior." *Journal of Financial and Quantitative Analysis* 36:431–449.

Hennessy, C., and T. Whited. 2007. "How Costly is External Financing? Evidence from a Structural Estimation." *Journal of Finance* 62:1705–1745.

Holmstrom, B., and J. Tirole. 1997. "Financial Intermediation, Loanable Funds, and the Real Sector." *Quarterly Journal of Economics* 112:663–691.

Hong, H., N. Wang, and J. Yang. 2021. "Welfare Consequences of Sustainable Finance." Working Paper.

Hugonnier, J., S. Malamud, and E. Morellec. 2015. "Capital Supply Uncertainty, Cash Holdings, and Investment." *Review of Financial Studies* 28:391–445.

IPCC. 2022. "Sixth Assessment Report, Climate Change 2022." IPCC Report, Working Group III.

Jensen, M. 1986. "Agency Costs of Free Cash Flow, Corporate Governance, and Takeovers." *American Economic Review* 76:323–329.

Kaldorf, M., and M. Shi. 2024. "Do Firm Credit Constraints Impair Climate Policy?" Working Paper.

Kim, S., T. Li, and Y. Wu. 2024. "Carbon Offsets: Decarbonization or Transition-Washing?" Working Paper.

Kruse, T., M. Mohnen, P. Pope, and M. Sato. 2020. "Green Revenues, Profitability and market valuation: Evidence from global firm level dataset." Working Paper.

Kumar, M. 2024. "Getting Dirty Before You Get Clean: Institutional Investment in Fossil Fuels and the Green Transition." Working Paper.

Landier, A., and S. Lovo. 2024. “ESG Investing: How To Maximize Impact.” Working Paper.

Lanteri, A., and A. Rampini. 2023. “Financing the Adoption of Clean Technology.” Working Paper.

Malamud, S., and F. Zucchi. 2019. “Liquidity, Innovation, and Endogenous Growth.” *Journal of Financial Economics* 132:519–541.

Meier, J.M., H. Servaes, J. Wei, and S. Xiao. 2023. “Do Consumers Care about ESG? Evidence from Barcode-Level Sales Data.” Working Paper.

Morse, A., and P. Sastry. 2024. “The Economics of Net Zero Banking.” Working Paper.

Noh, D., S. Oh, and J. Song. 2023. “Unpacking the Demand for Sustainable Equity Investing.” Working Paper.

Oehmke, M., and M. Opp. 2024. “A Theory of Socially Responsible Investment.” *Review of Economic Studies,* Forthcoming.

Opler, T., L. Pinkowitz, R. Stulz, and R. Williamson. 1999. “The Determinants and Implications of Corporate Cash Holdings.” *Journal of Financial Economics* 52:3–46.

Pacca, L., A. Antonarakis, P. Schroder, and Antoniades. 2020. “The Effect of Financial Crises on Air Pollutant Emissions: An Assessment of the Short vs. Medium-term Effects.” *Science of the total Environment* 698:133614.

Peters, G., G. Marland, C. Le Quere, T. Boden, J. Canadell, and M. Raupach. 2012. “Rapid Growth of CO2 Emissions After the 2008–2009 Global Financial Crisis.” *Nature Climate Change* 2:2–4.

Sheldon, T. 2017. “Asymmetric Effects of the Business Cycle on Carbob Dioxide Emissions.” *Energy Economics* 61:289–297.

Shi, M., S. Zhou, and C. Meinerding. 2023. “Do Past Emissions Affect Firms’ Supply of Green Technologies: Evidence from Technology-Related Job Postings.” Working Paper.

Zucchi, F. 2024. “The Real Effects of Financing and Trading Frictions.” *Journal of Financial and Quantitative Analysis,* Forthcoming.